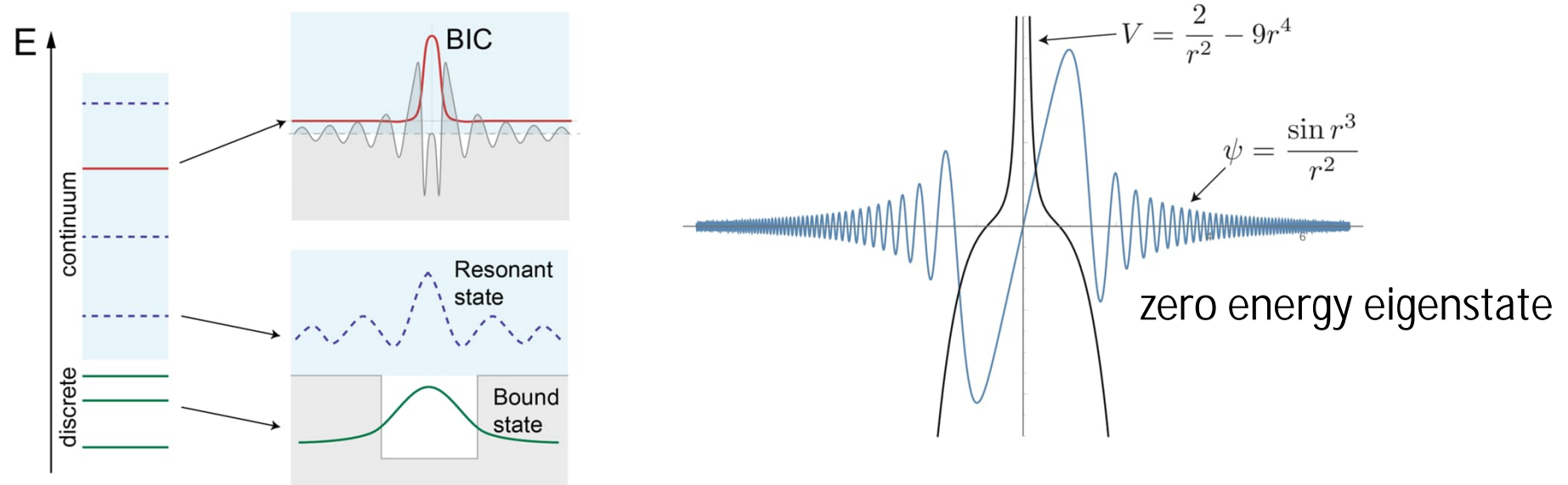


Bound state In the Continuum (BIC)

J. v. Neumann and E. P. Wigner, "Über merkwürdige diskrete Eigenwerte, Physikalische Zeitschrift 30, 465-467 (1929).

Originally a theoretical concept of QM (solved by v. Neumann and Wigner):

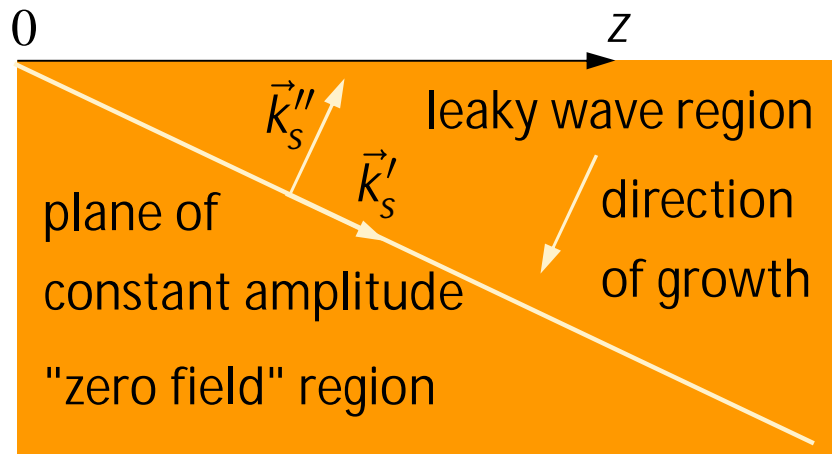
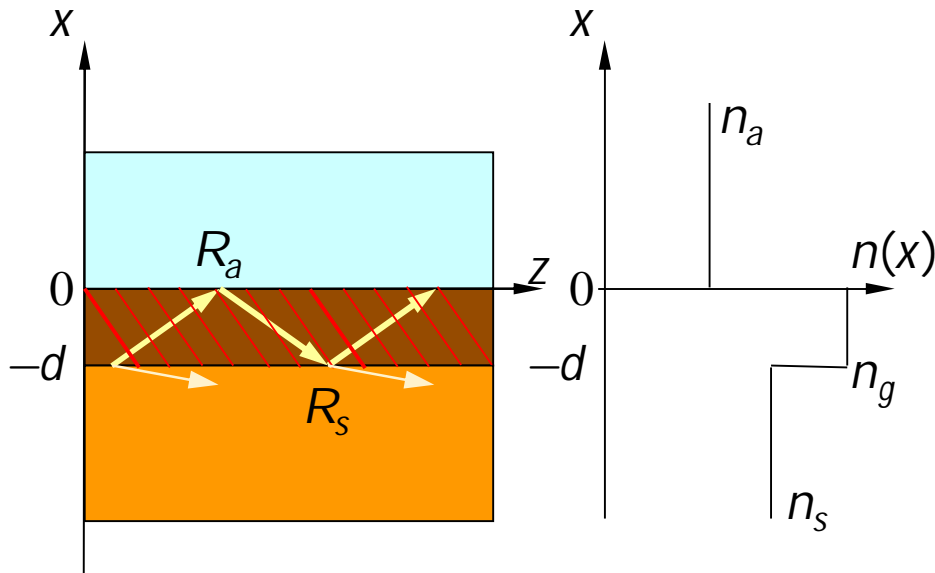


First observed in acoustics, later *many occurrences* were found in photonic structures (gratings, resonators, *waveguiding structures*).

BIC – bound mode *not coupled* to radiation continuum: *real* propagation constant (or frequency);
q-BIC (quasi-BIC) – quasi-bound mode weakly coupled to radiation continuum: *complex* propagation constant (or frequency) with *small imaginary part*

Leaky mode in a planar waveguide

Leaky modes in standard planar waveguides



$$R_a R_s e^{2ik_0 \gamma_g d} = 1$$

$$\text{Re}\{N\} < n_s : |R_s| < 1$$

N is a complex number :

$$N = N' + iN'', \quad N'' > 0.$$

Wave in the substrate: $e^{ik_0(Nz - \gamma_s x)}$,

$$\gamma_s = \sqrt{n_s^2 - N^2} = \gamma_s' + i\gamma_s'', \quad \gamma_s'' < 0;$$

$$e^{ik_0(Nz - \gamma_s x)} = e^{ik_0(N'z - \gamma_s' x)} e^{-k_0(N''z + \gamma_s'' x)},$$

$$e^{k_0 \gamma_s'' x} = e^{-k_0 |\gamma_s''| x}$$

The wave is **growing** for $x \rightarrow -\infty$!!!

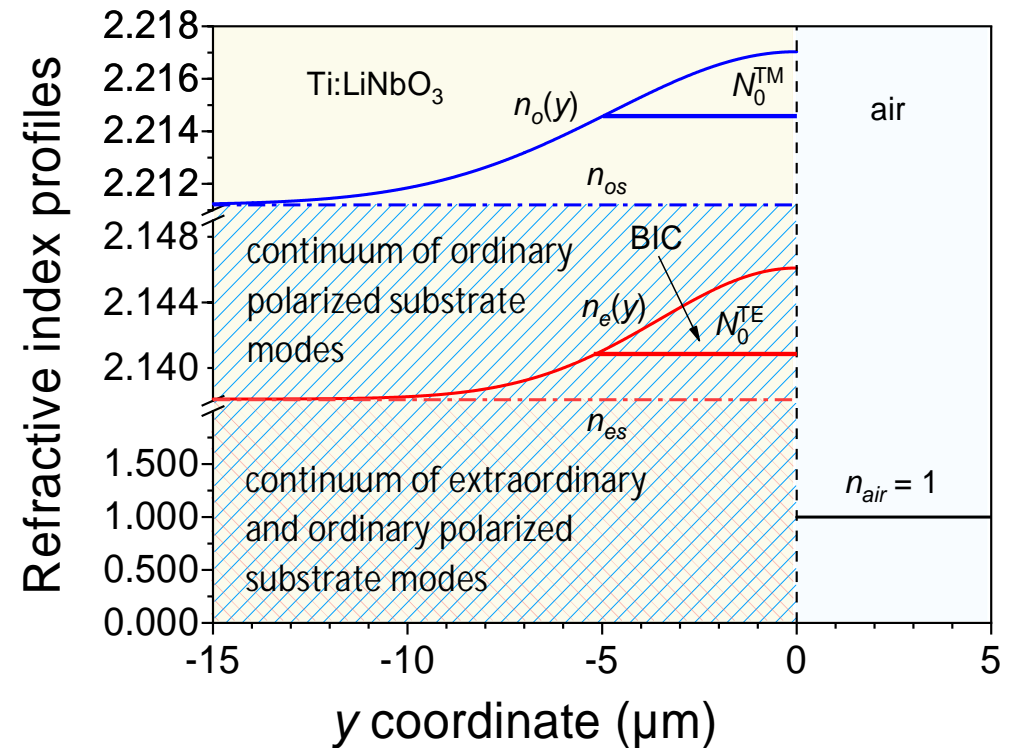
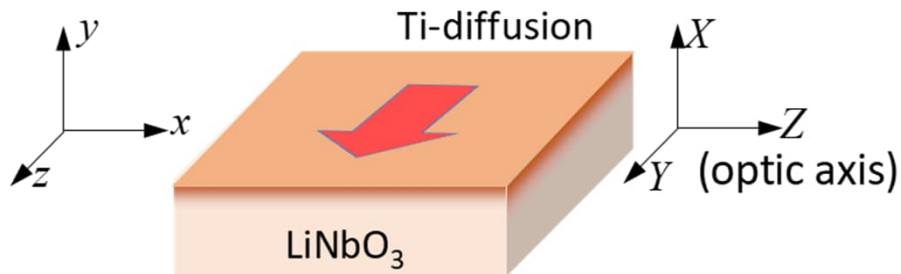
Leaky modes are **improper modes**

of the eigenvalue equation;

they can be represented by

a **superposition of radiation modes**.

BICs in planar Ti:LiNbO₃ and (A)PE LiNbO₃ waveguides



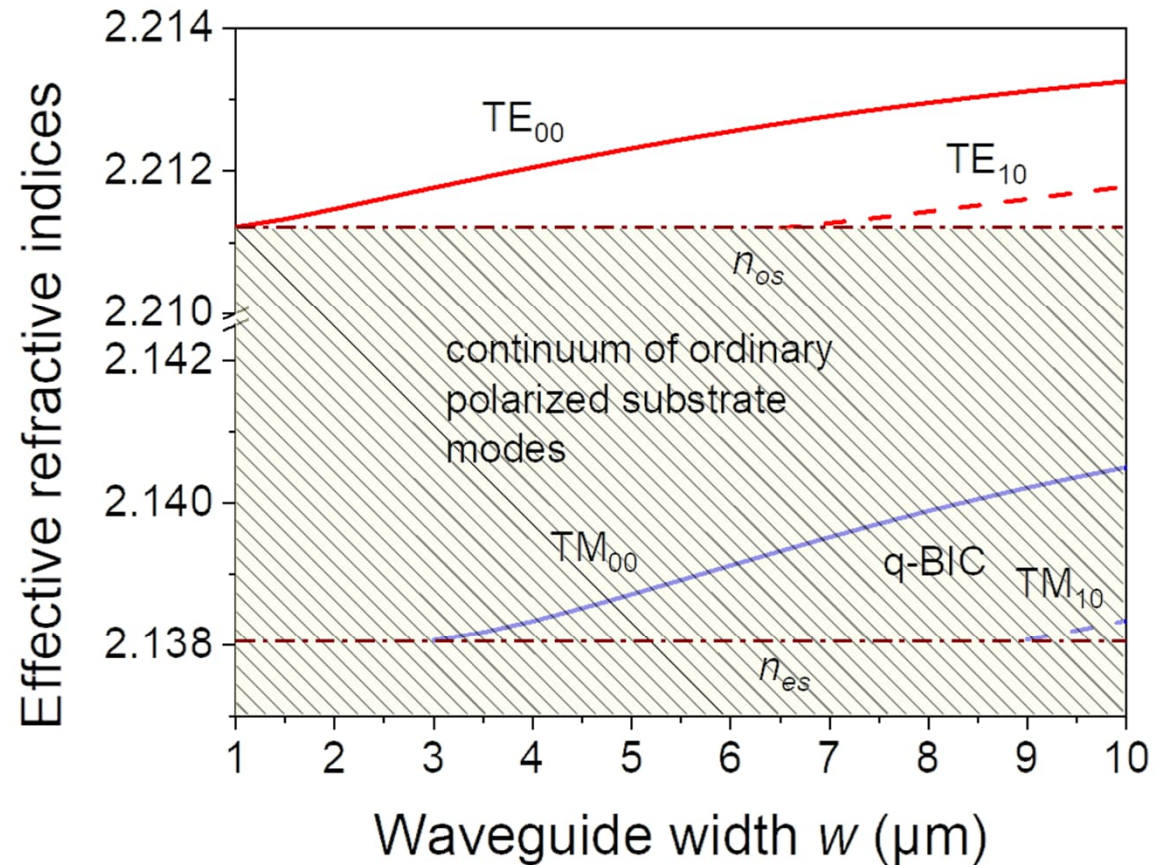
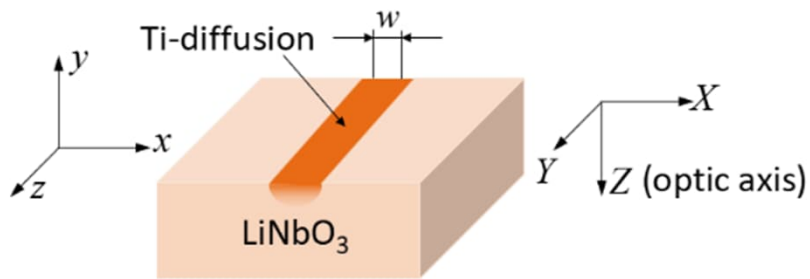
TE modes of Z-cut Ti:LiNbO₃ planar waveguide, propagating in Y direction, have only a single component of the electric field intensity E_z which "sees" the extraordinary refractive index. Such modes thus cannot be coupled with ordinary polarized radiation modes.

Since $n_o > n_e$, the propagation constants of TE modes are embedded into the continuum of propagation constants of ordinary polarized radiation modes, and form thus *pure BICs*.

Similar situation takes place for TM modes of Z-cut Ti:LiNbO₃ waveguides propagating in an arbitrary direction.

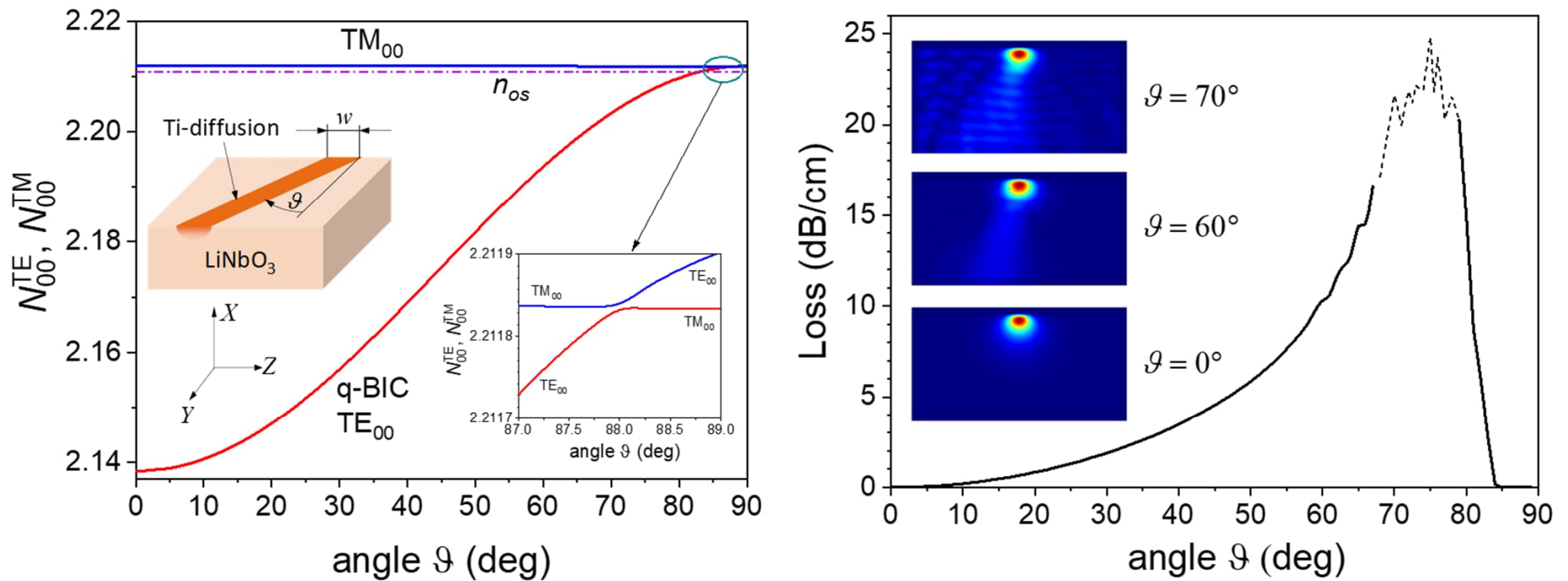
(A)PE waveguides behave very similarly.

q-BICs in Z-cut channel Ti:LiNbO₃ and (A)PE LiNbO₃ waveguides



Modes of *channel* waveguides have generally all 6 components of electric and magnetic fields nonzero. It means that, *in principle*, the (quasi-)TM modes of Z-cut Ti:LiNbO₃ can be coupled with the ordinary polarized continuum via the minority electric field component E_x . Numerical simulations using COMSOL Multiphysics anisotropic mode solver revealed very low radiation loss into the substrate of the order of 10^{-3} dB/cm; such modes can be thus classified as *low-loss q-BICs*.

q-BICs in X-cut channel Ti:LiNbO₃ and (A)PE LiNbO₃ waveguides

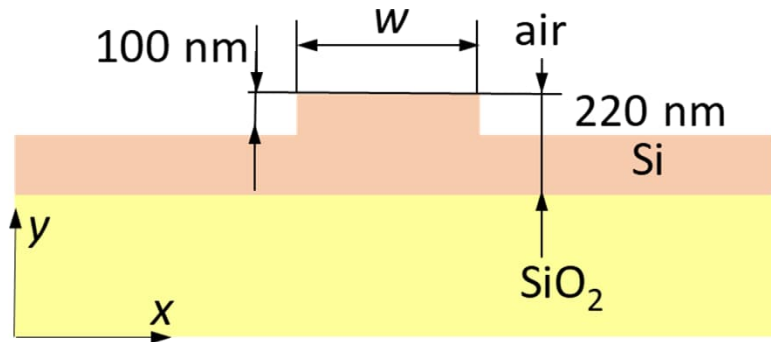


TE modes of X-cut channel waveguides directed in an arbitrary direction exhibit radiation into substrate due to coupling with ordinary polarized continuum.

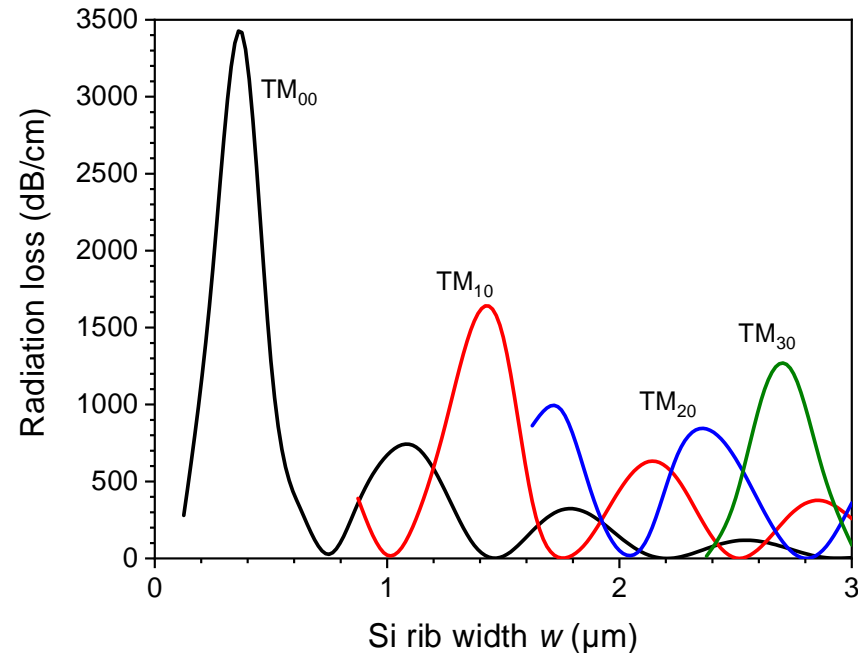
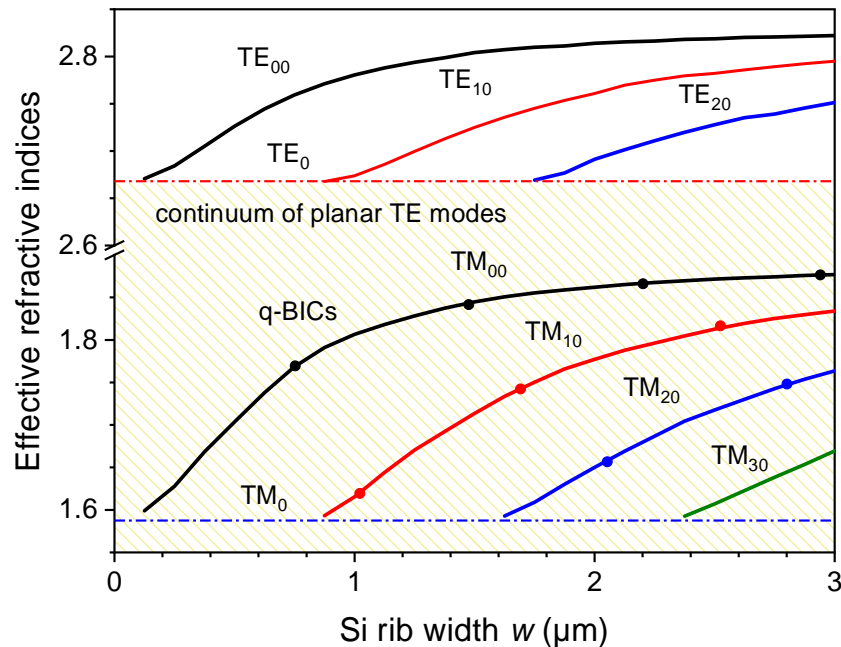
Radiation loss depends on the direction of propagation, as it is shown in the insets in the right graph. These modes are thus *lossy q-BICs*.

For the direction of propagation close to the optic axis Z, a *mode conversion* takes place, as it is shown in the inset of the left graph. These modes are pure bound (non-BIC) modes.

q-BICs in SOI ridge waveguides

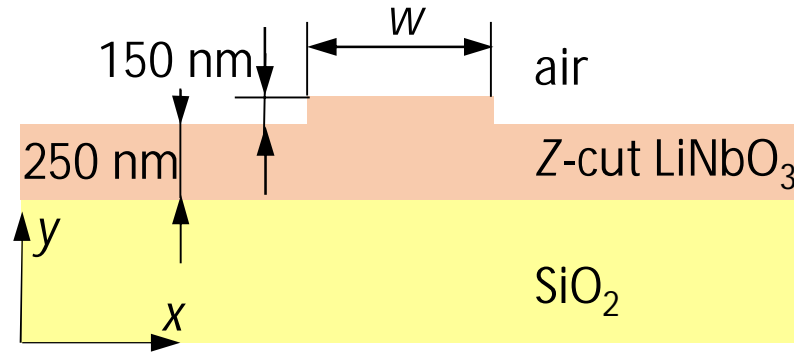


TM modes of the channel ridge waveguides can be coupled with the TE-polarized modes of the residual *planar* waveguide, creating thus q-BICs.



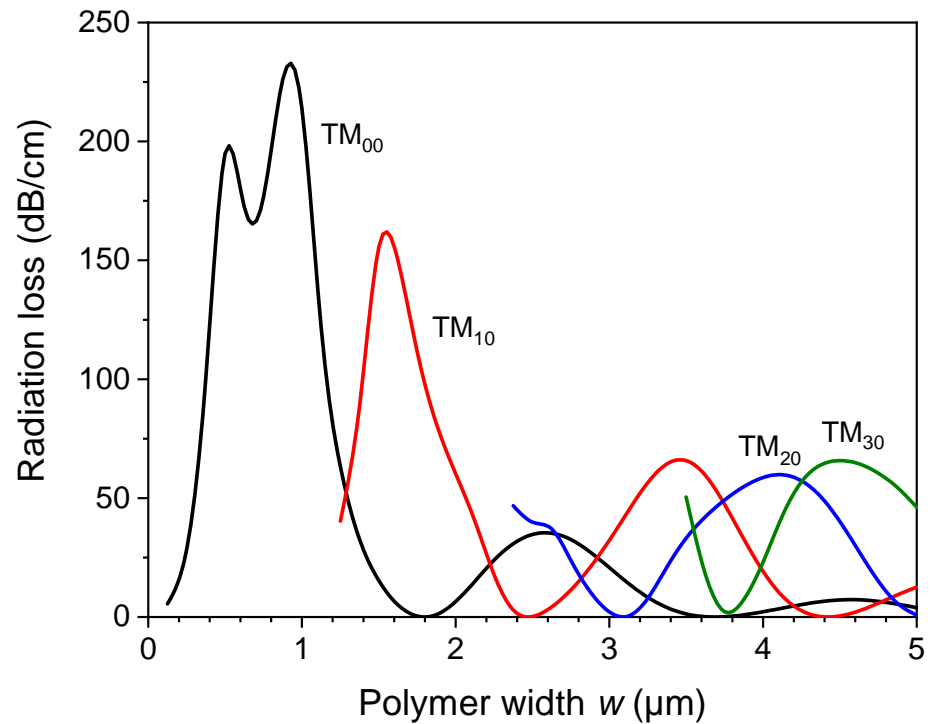
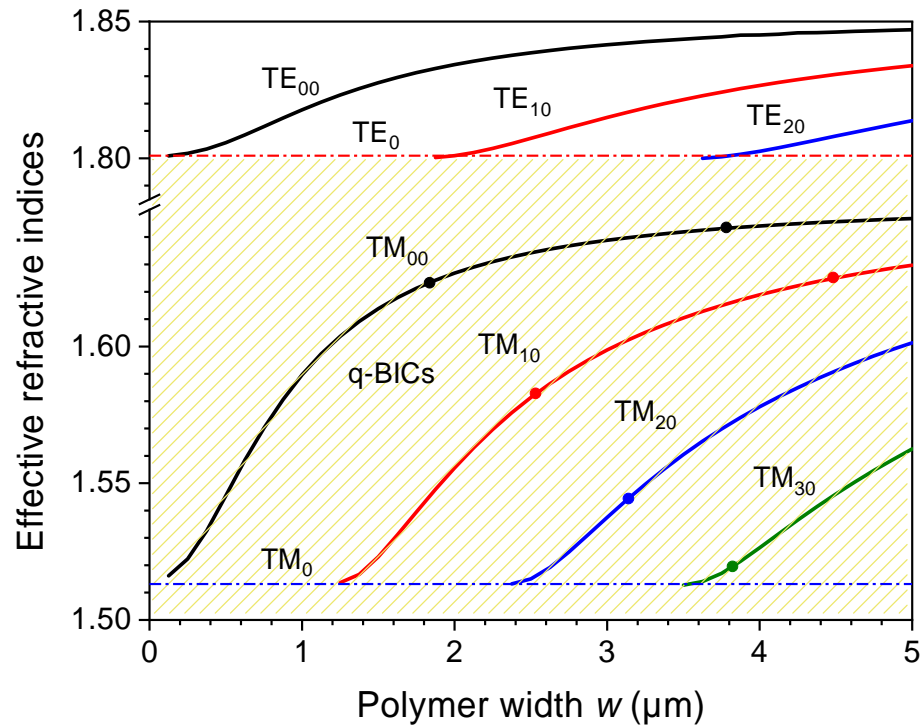
TE modes are standard bound (non-BIC) modes, while TM modes are lossy q-BICs, except for “magic widths”, for which the loss is vanishing and modes become q-BICs.

q-BICs in Z-cut LNOI ridge waveguides

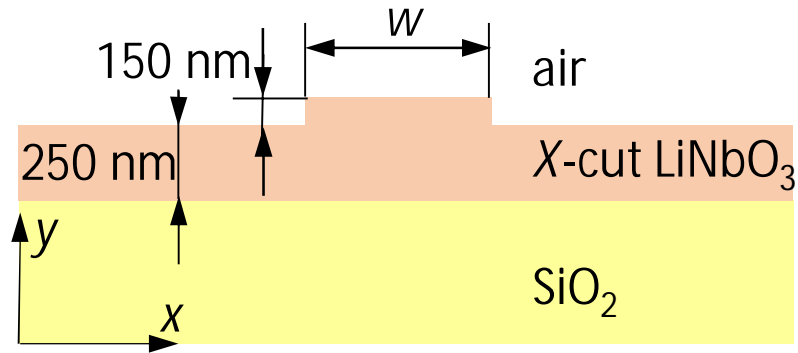


The Z-cut exhibits rotational symmetry, the properties of waveguides are thus independent of their direction in the XY plane

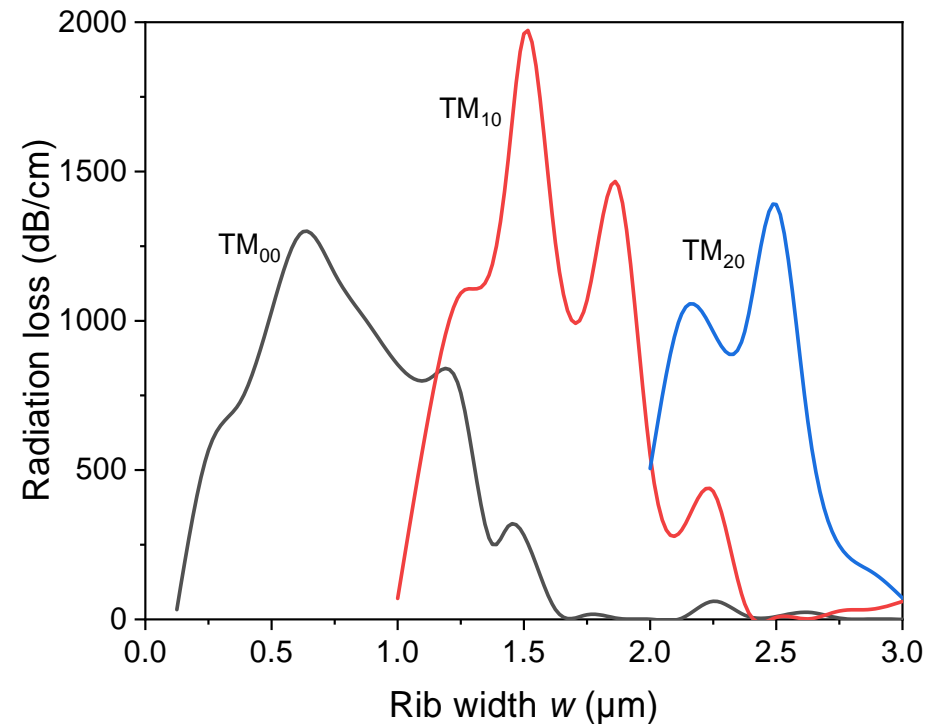
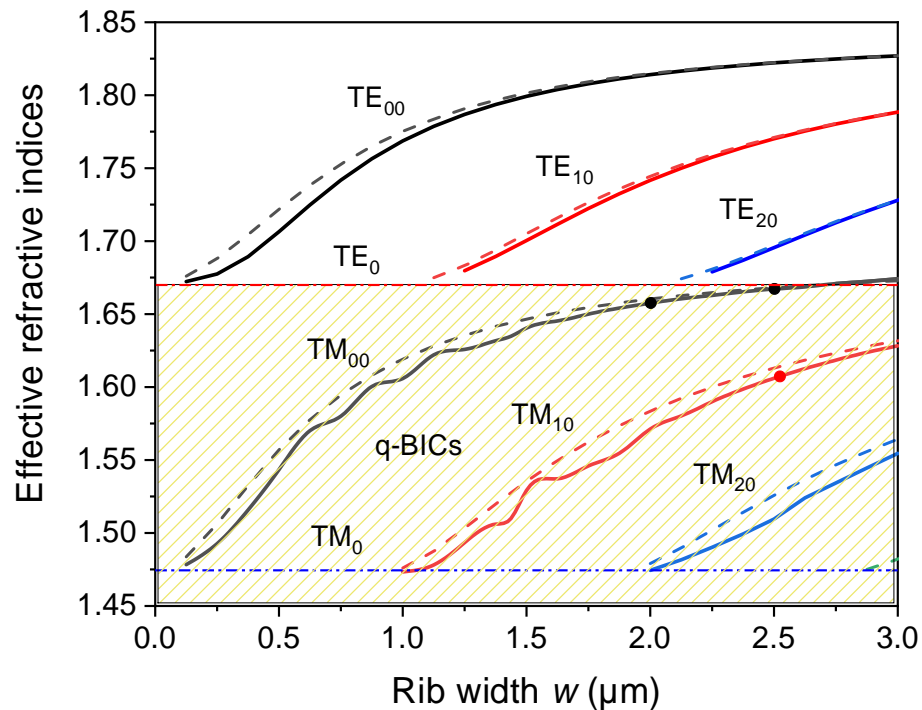
TM modes are preferentially extraordinary polarized and support q-BICs; for „magic widths“, the waveguides support BICs.



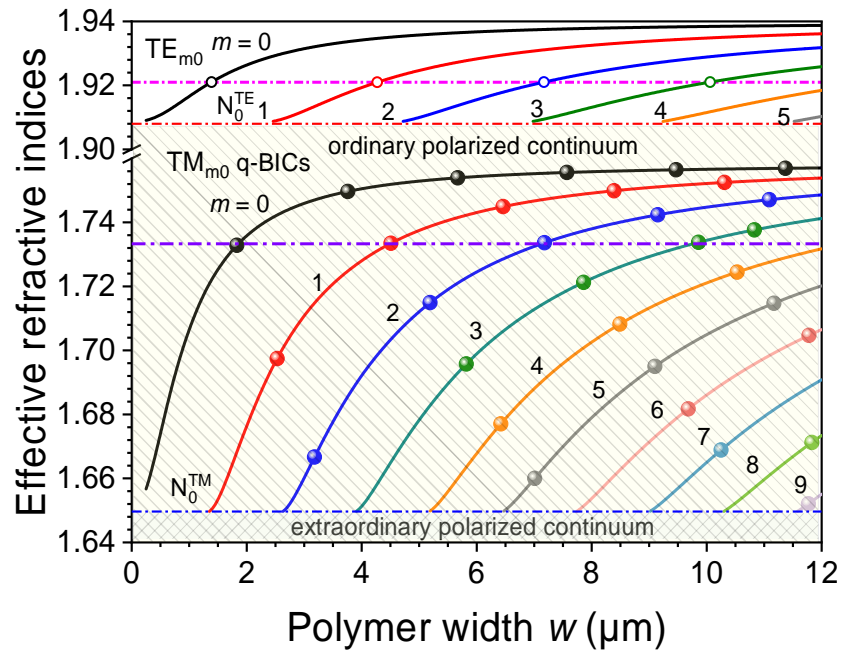
q-BICs in X-cut Y-propagating LNOI ridge waveguides



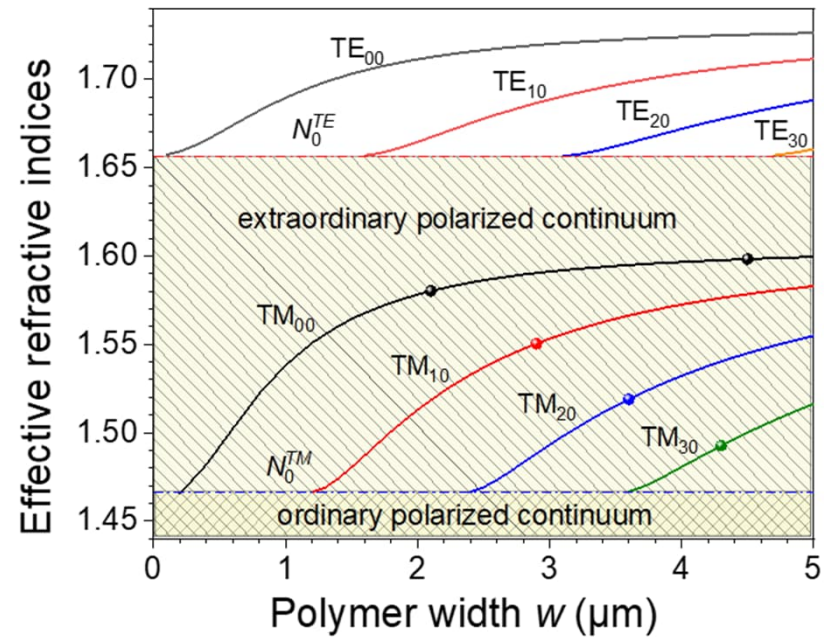
Even though the TM modes are ordinary polarized, their effective indices are lower due to the waveguide birefringence. The radiation loss is very strong.



q-BICs in low-index polymer-loaded LNOI waveguides

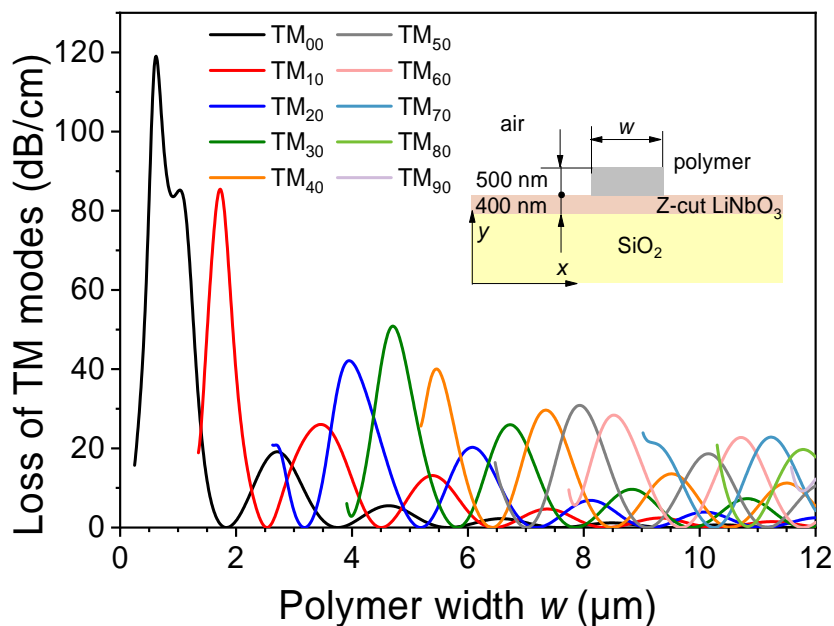


Z-cut LNOI



X-cut LNOI

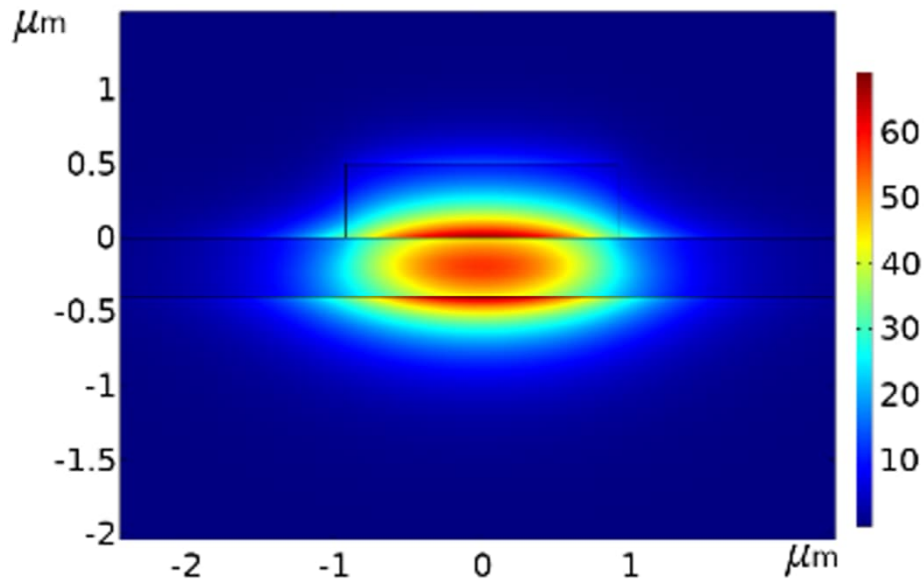
LN thickness 240 nm
poly. thickness 500 nm,
 $n_{poly} = 1.5429$ (ZEP520A)



- Z. J. Yu, X. Xi, J. W. Ma, H. K. Tsang, C. L. Zou, and X. K. Sun, "Photonic integrated circuits with bound states in the continuum," *Optica* 6, 1342-1348 (2019).
- Z. Yu, Y. Tong, H. K. Tsang, and X. Sun, "High-dimensional communication on etchless lithium niobate platform with photonic bound states in the continuum," *Nat. Commun.* 11, 2602 (2020).
- J. Zhang, B. Pan, W. Liu, D. Dai, and Y. Shi, "Ultra-compact electro-optic modulator based on etchless lithium niobate photonic crystal nanobeam cavity," *Opt. Express* 30, 20839 (2022).

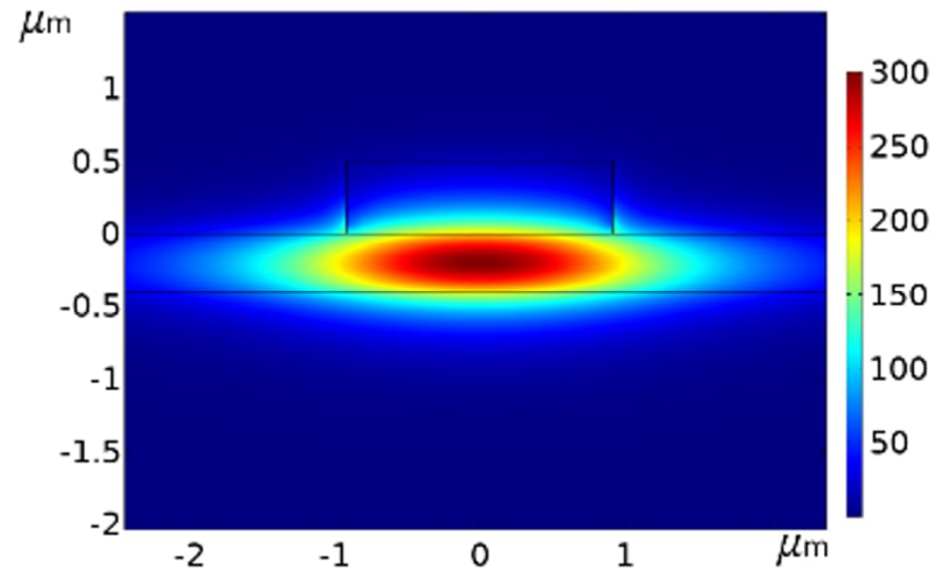
Electric field intensity distribution of TM and TE modes in polymer-loaded waveguides

TM₀₀ mode (BIC)



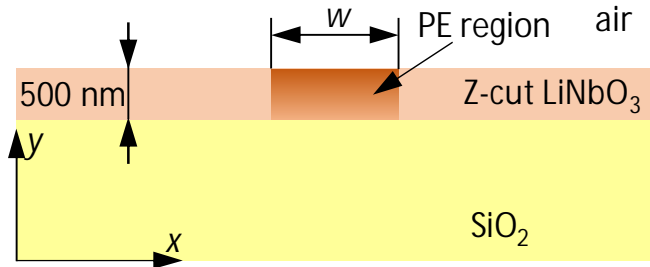
LN thickness 400 nm,
polymer thickness 500 nm,
waveguide width 1.83 μm
("magic width" of TM BIC mode)

TE₀₀ mode (standard, non-BIC)

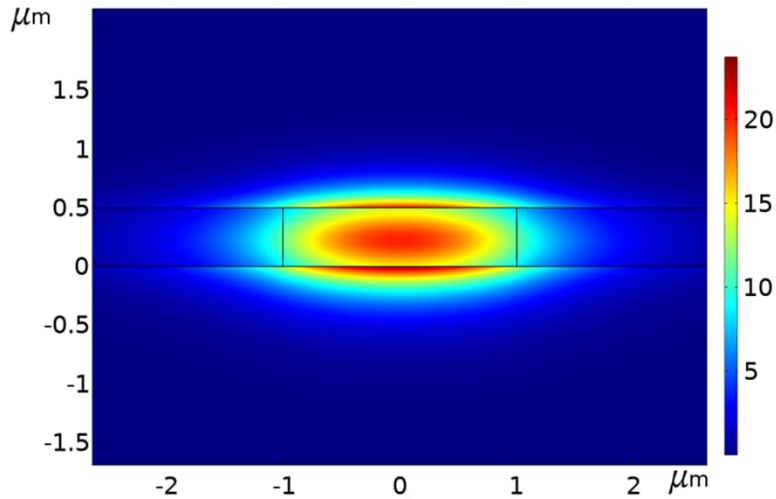


TE mode is more strongly localized
within the LiNbO₃ crystal slab;
it is more suitable for electro-optic
modulation, despite its

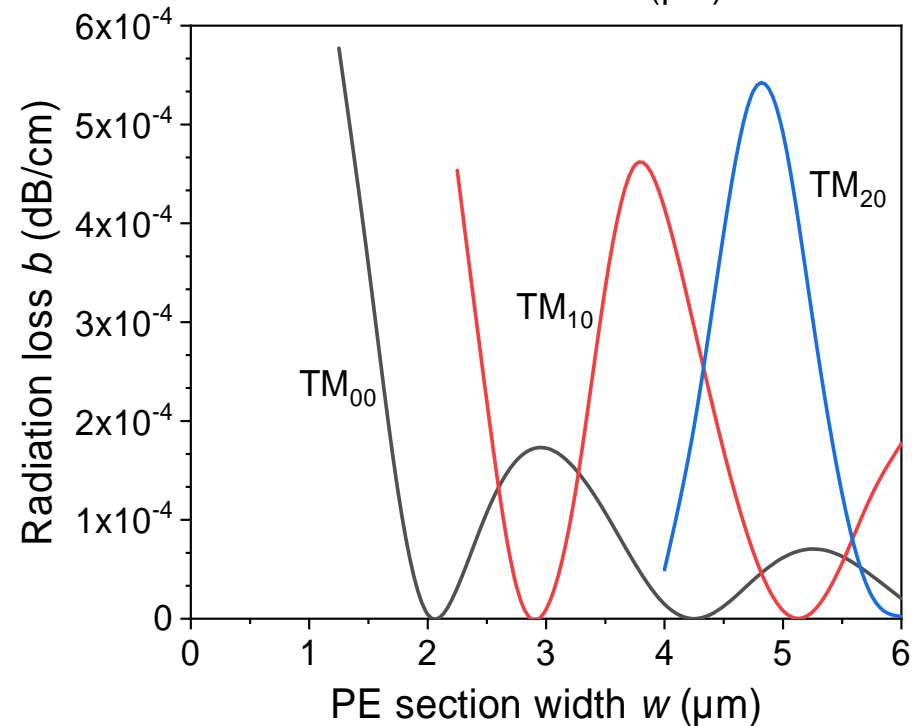
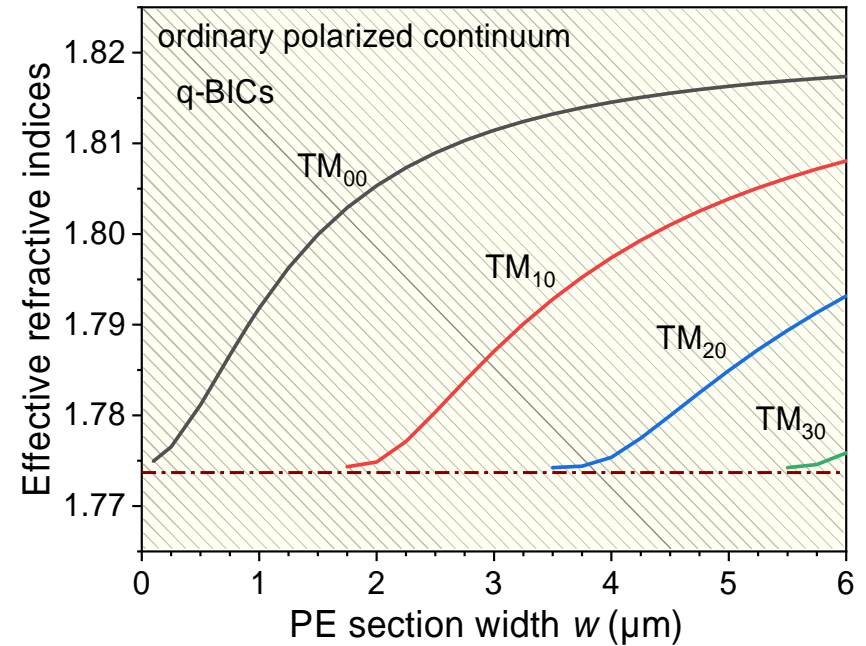
q-BICs in proton-exchanged LNOI waveguides



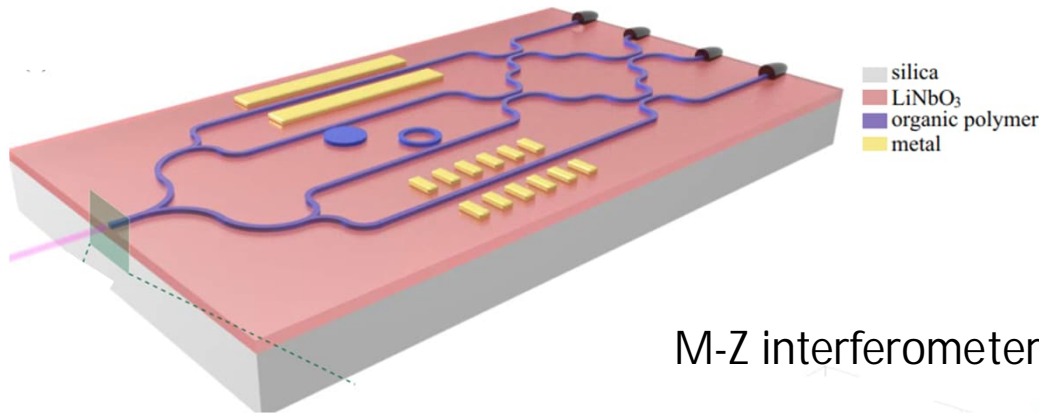
$$\Delta n_e \leq 0.1, \quad \Delta n_o \simeq -\frac{1}{3} \Delta n_e < 0.$$



“Etchless” LNOI technology;
 waveguides support propagation
 of extremely low-loss q-BIC(TM) modes;
 potentially advantageous platform
 for integrated-photonics electro-
 optic devices (fabrication process has
 to be improved to mitigate the scattering loss)



Some recent applications of q-BICs in polymer-loaded LNOI waveguides - I

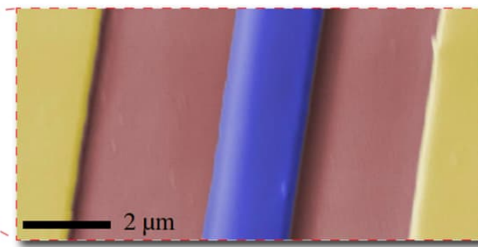
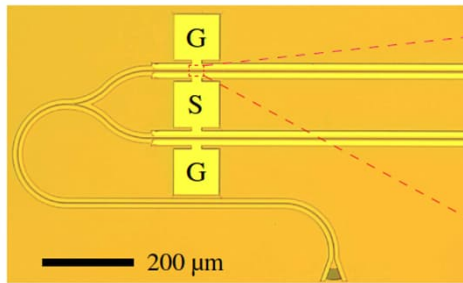


Z. J. Yu, X. Xi, J. W. Ma, H. K. Tsang, C. L. Zou, and X. K. Sun, "Photonic integrated circuits with bound states in the continuum," *Optica* 6, 1342-1348 (2019).

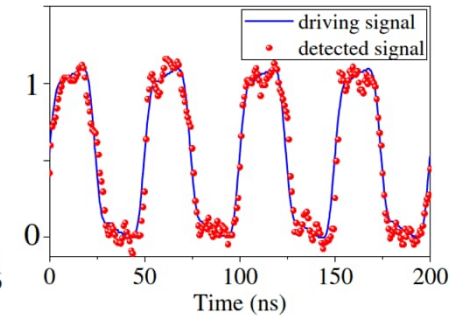
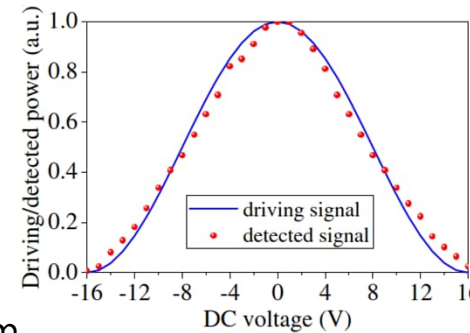
X-cut LNOI, 300 nm LN, 500 nm polymer

M-Z interferometer:

$$\Delta n_e = -\frac{1}{2} n_e^3 r_{51} E_y$$



$$V_\pi \approx 16 \text{ V}, L_e = 5 \text{ mm}.$$



J. Č. and J. Petráček, "Photonic integrated circuits with bound states in the continuum: comment," *Optica* 9, 681-682 (2022).

$$\Delta N_{eff} \approx -\frac{Y_0}{2} \frac{\iint_S [n_o^4 \gamma_{22} (e_x^2 + e_y^2) + 2n_o^4 n_e^4 \gamma_{51} e_y e_z] E_y dS}{\iint_S (e_y h_z - e_z h_y) dS} \approx \frac{Y_0}{2} \frac{\iint_S n_o^4 \gamma_{22} (e_x^2 + e_y^2) E_y dS}{\iint_S (e_y h_z - e_z h_y) dS};$$

$V_\pi \approx 260 \text{ V}.$

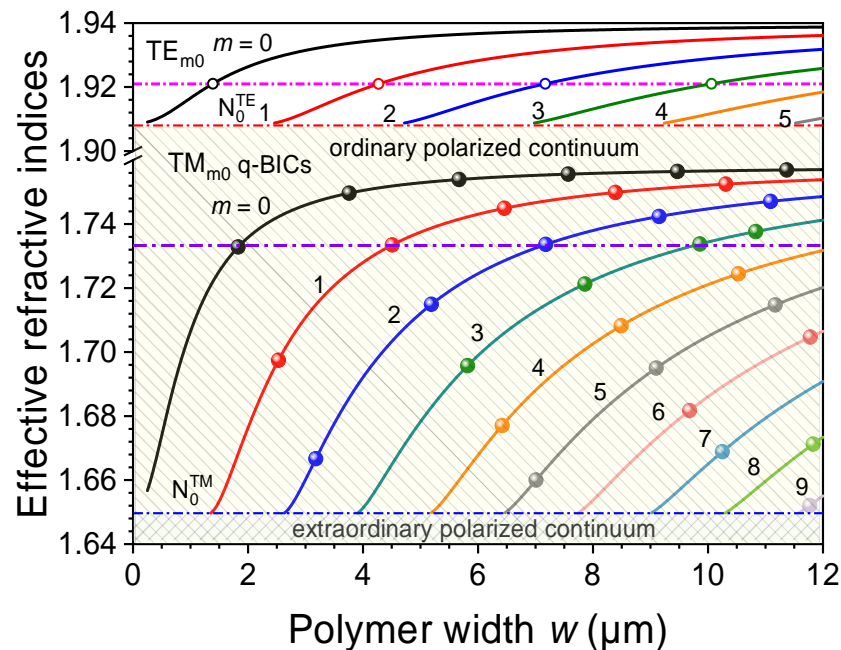
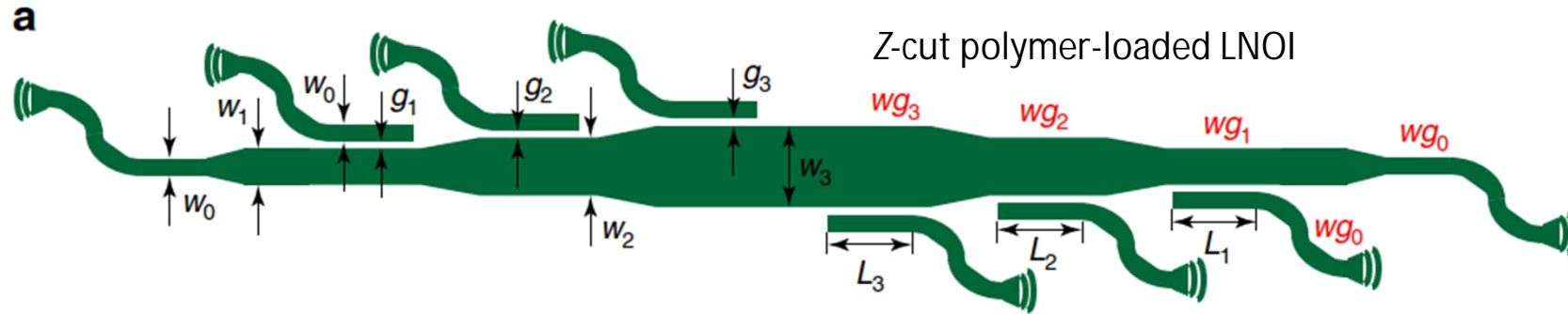
Z. Yu, X. Xi, J. Ma, H. K. Tsang, C.-L. Zou, and X. Sun, "Photonic integrated circuits with bound states in the continuum: erratum," *Optica* 9, 683 (2022):

$$\Delta n_e = -\frac{1}{2} n_e^3 r_{51} E_y \tan \gamma, L_e = 50 \text{ mm (???)}$$

γ ... angle of rotation of the optic axis due to the applied field; however, this is not a *linear* EO effect, and $\gamma \approx 10^{-3}$ (!!!)

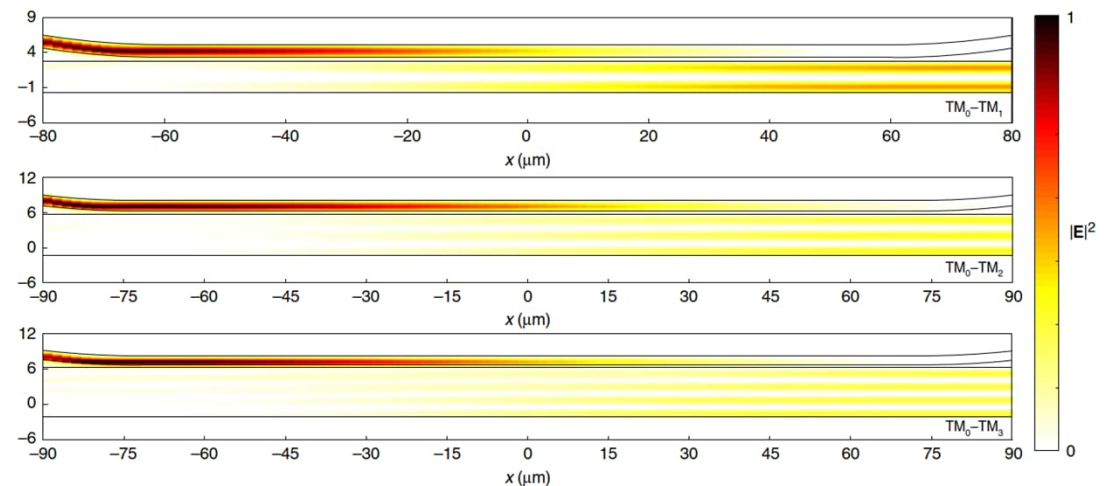
Some recent applications of q-BICs in polymer-loaded LNOI waveguides - II

Z. Yu, Y. Tong, H. K. Tsang, and X. Sun, "High-dimensional communication on etchless lithium niobate platform with photonic bound states in the continuum," Nat. Commun. 11, 2602 (2020).



Effective refractive indices of q-BIC modes TM_{00} , TM_{10} , TM_{20} , and TM_{30} are nearly equal. These modes are used in *asymmetric DC couplers* (application for *mode-division* multiplexing).

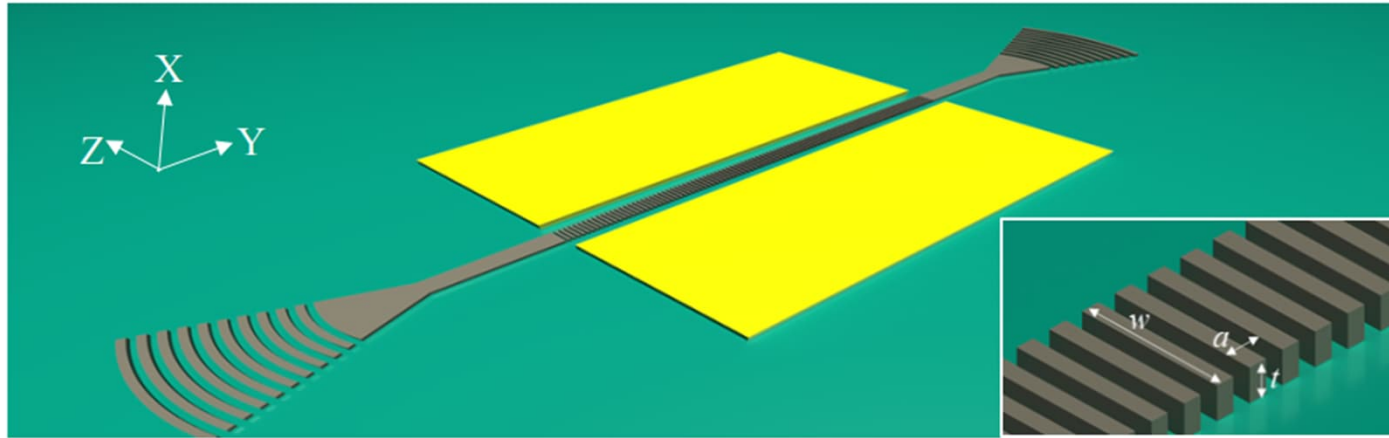
Simulations of TM_{00} to TM_{m_0} mode coupling:



The proximity of the second waveguide affects the phase synchronism \rightarrow the waveguide widths have to be modified, and the same effect can be obtained with (non-BIC) TE_{m_0} modes, without limitation to "magic widths"...

Some recent applications of q-BICs in polymer-loaded LNOI waveguides - III

J. Zhang, B. Pan, W. Liu, D. Dai, and Y. Shi, "Ultra-compact electro-optic modulator based on etchless lithium niobate photonic crystal nanobeam cavity," Opt. Express 30, 20839 (2022).

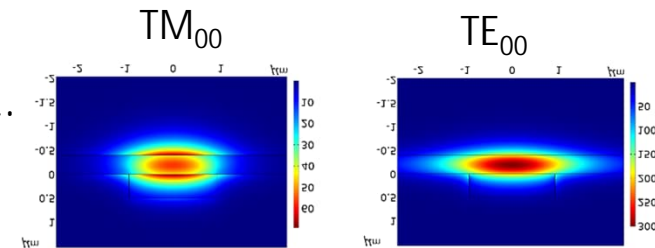
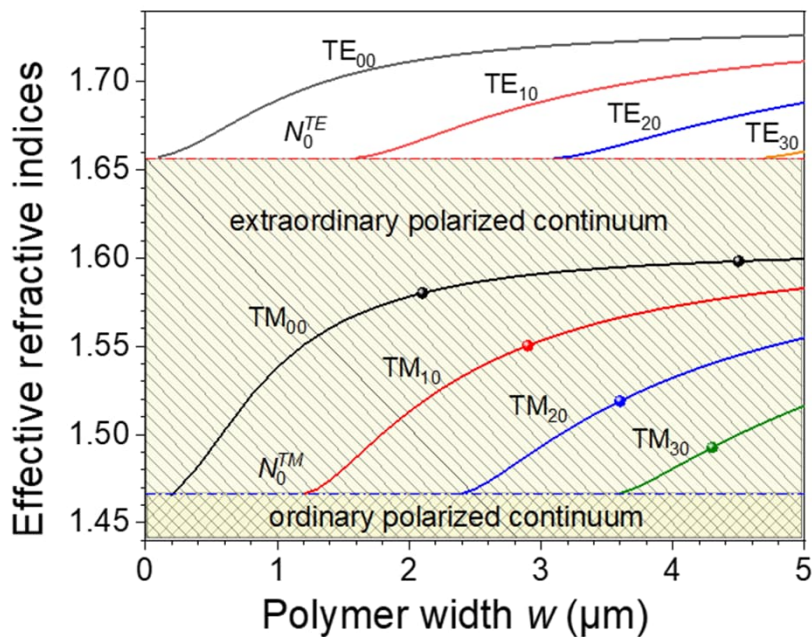


X-cut polymer-loaded LNOI; SWG waveguide close to the band gap to increase the EO interaction;
Low-loss TM_{00} (q-)BIC mode is used, the electro-optic effect makes use of the coefficient r_{13} :

$$\Delta n_o \doteq -\frac{1}{2} n_o^3 r_{13} E_z.$$

The application of TE_{00} mode would remove the restriction to the waveguide "magic width" and allow for application of about 3× stronger electro-optic coefficient r_{33} :

$$\Delta n_e \doteq -\frac{1}{2} n_e^3 r_{33} E_z.$$



Together with stronger confinement of the TE_{00} mode in the LN crystal, the implementation of the TE_{00} mode would help reduce the half-wave voltage of the modulator approximately by a factor of 7.