Gain-Loss photonic structures:

formal analogy with quantum-mechanical

structures with $\mathcal{PT}$ symmetry/breaking;

(non-Hermitean theory)

Introduction: Early papers on gain-loss structures (without any comment on PT symmetry/breaking)

Brief history of asymmetric complex grating-assisted coupler:

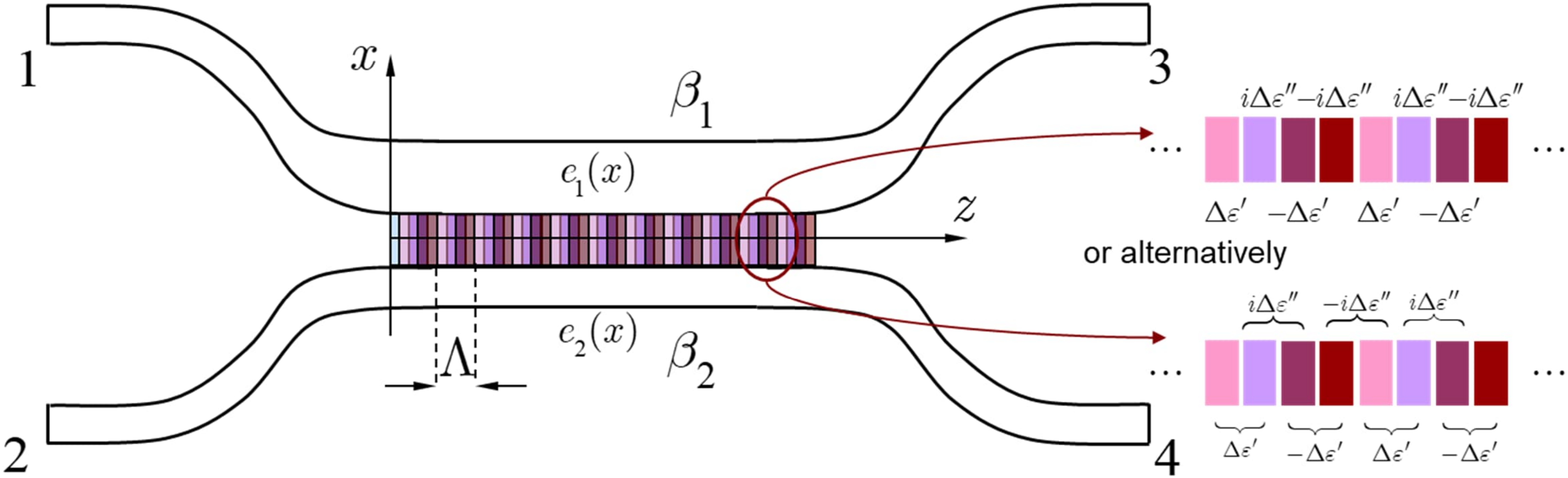
Basic theory:

L. Poladian, *Phys. Rev. E,* 54, 2963-2975, (1996).
M. Greenberg and M. Orenstein, *Optics Express*, 12, 4013-4018, 2004.
M. Greenberg and M. Orenstein, *Optics Letters*, 29, pp. 451-453, 2004.
M. Greenberg and M. Orenstein, *IEEE J. Quantum Electron.*, 41, 1013-1023, 2005.

Application proposals:

M. Greenberg and M. Orenstein, *Phot. Tech. Lett*., 17, 1450-1452, 2005. (add mux)
M. Kulishov et al, *Optics Express*, 13, 3567-3578, 2005 (light trapping in ring resonator)
…

ASYMMETRIC COMPLEX GRATING COUPLER



Grating-assisted directional coupler using ***asymmetric complex grating***

$$E_y(x,z) \approx A_1(z)e_1(x)\exp(i\beta_1 z) + A_2(z)e_2(x)\exp(i\beta_2 z);$$

complex, periodic in z

$$\frac{dA_1(z)}{dz} \cong i\kappa_{11}(z)A_1(z) + i\kappa_{12}(z)e^{-i(\beta_1-\beta_2)z}A_2(z),$$

$$\frac{dA_2(z)}{dz} \cong i\kappa_{21}(z)e^{i(\beta_1-\beta_2)z}A_1(z) + i\kappa_{22}(z)A_2(z),$$

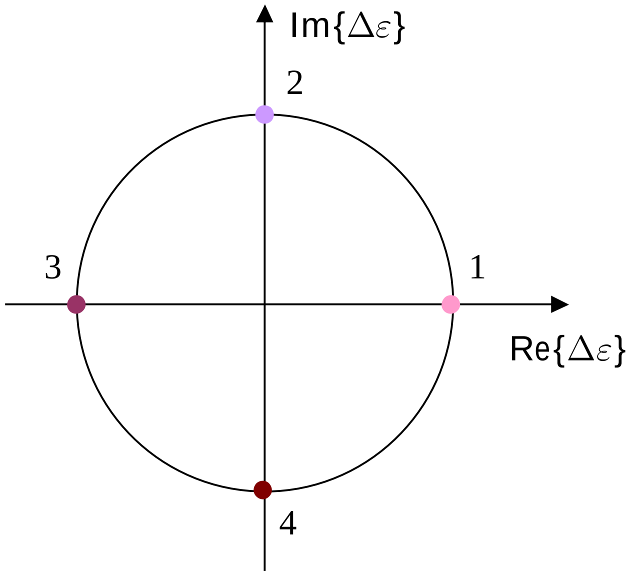
$$\kappa_{mn}(z) = \frac{k_0}{2}\iint_S \Delta\varepsilon(x,z)e_m(x)e_n(x)\,dS$$

$$= \kappa_{mn}^{(1)}e^{iKz} + \kappa_{mn}^{(2)}e^{2iKz} + \ldots, \quad K = 2\pi/\Lambda$$

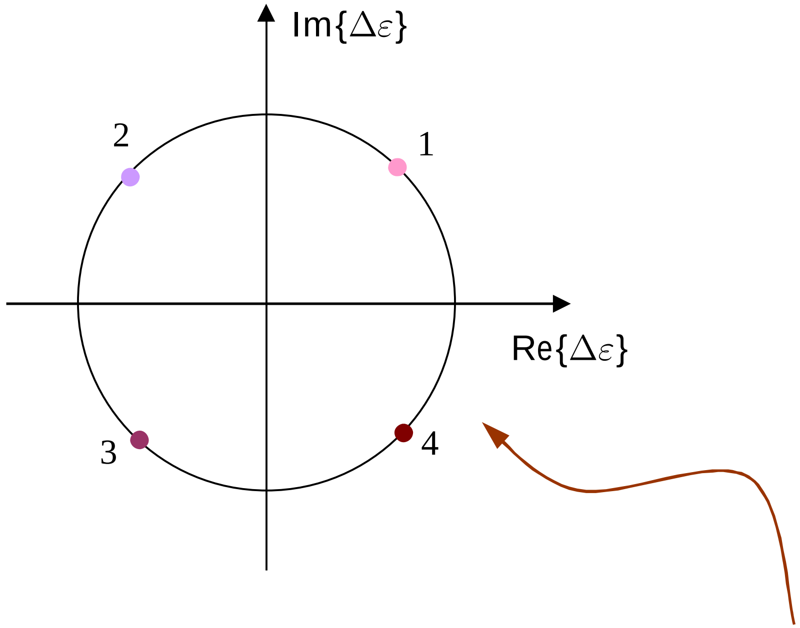
Fourier expansion contains only *positive exponentials* ("SSB modulation")

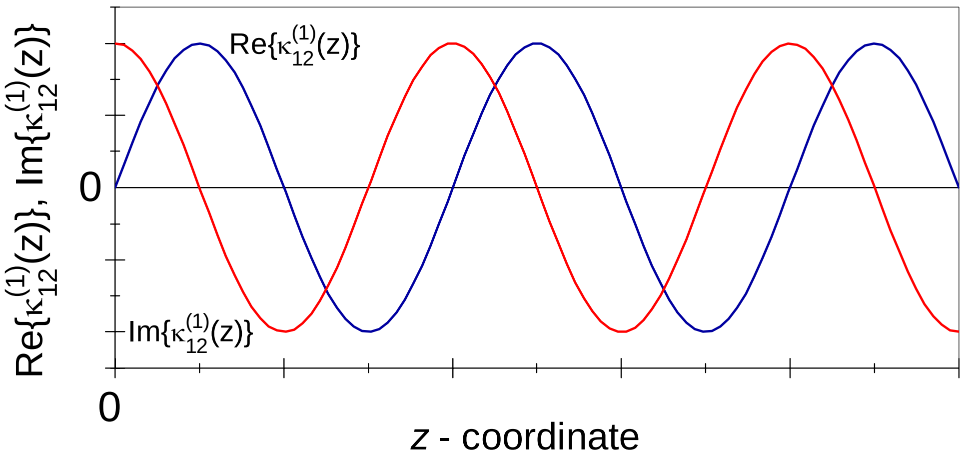
ASYMMETRIC COMPLEX GRATING

Complex permittivity perturbation in individual grating segments:



or





The second option seems technologically simpler since it requires only *two different values* of $\Delta\varepsilon'$ and $\Delta\varepsilon''$

SOLUTION OF COUPLED MODE EQUATIONS

Let us consider the following ideal case of the grating at synchronism,

$$\kappa_{11}(z) = 0, \quad \kappa_{12}(z) = \kappa_{12}^{(1)} \exp(iKz),$$
$$\kappa_{21}(z) = 0, \quad \kappa_{22}(z) = 0,$$
$$\Delta\beta = K - (\beta_1 - \beta_2) = 0.$$

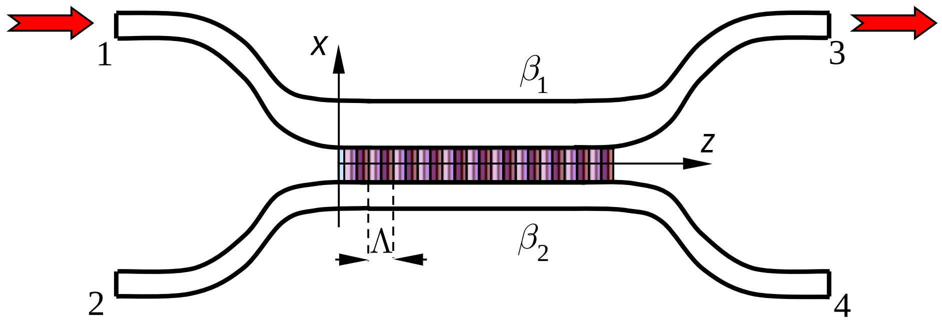
Then, the coupled equations read

$$\frac{dA_1(z)}{dz} \cong i\kappa_{12}^{(1)} A_2(z),$$
$$\frac{dA_2(z)}{dz} \cong 0.$$

For $A_1(0) \neq 0, \quad A_2(0) = 0$

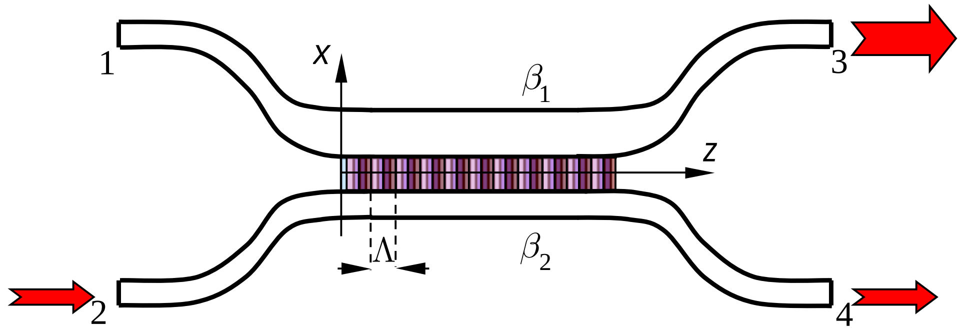
we get the solution

$$A_1(z) = A_1(0) = const., \quad P_1(z) = P_1(0),$$
$$A_2(z) = 0, \qquad P_2 = 0.$$



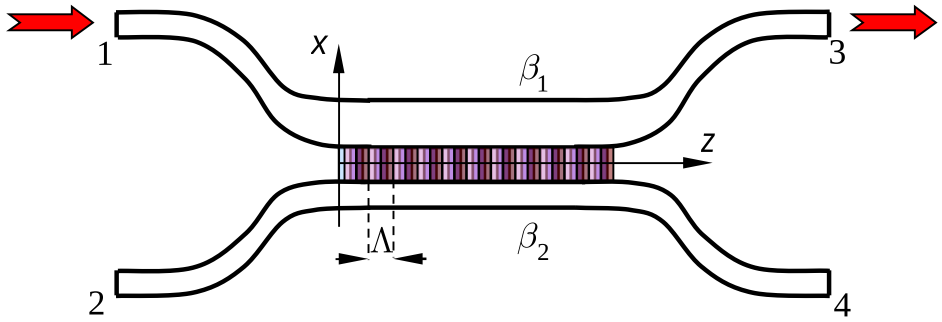
for $A_1(0) = 0, \quad A_2(0) \neq 0$

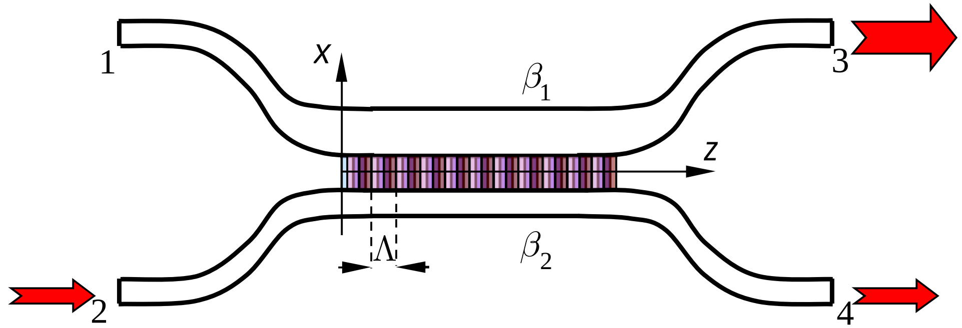
$$A_1(z) = i\kappa_{12}^{(1)} A_2(0) z, \qquad P_1(z) = \left|\kappa_{12}^{(1)}\right|^2 P_2(0) z^2,$$
$$A_2(z) = A_2(0) = const., \quad P_2(z) = P_2(0) = const.$$



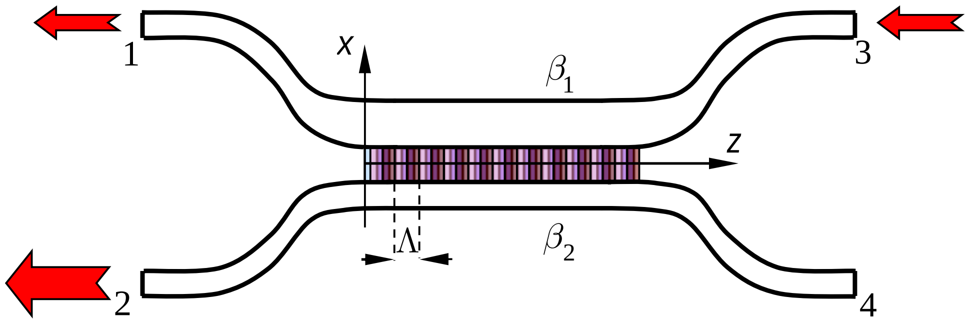
ACGC IS A RECIPROCAL DEVICE!

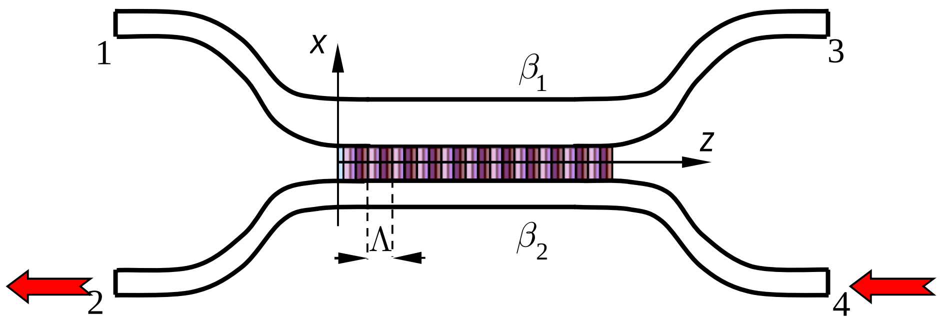
Forward propagation





Backward propagation

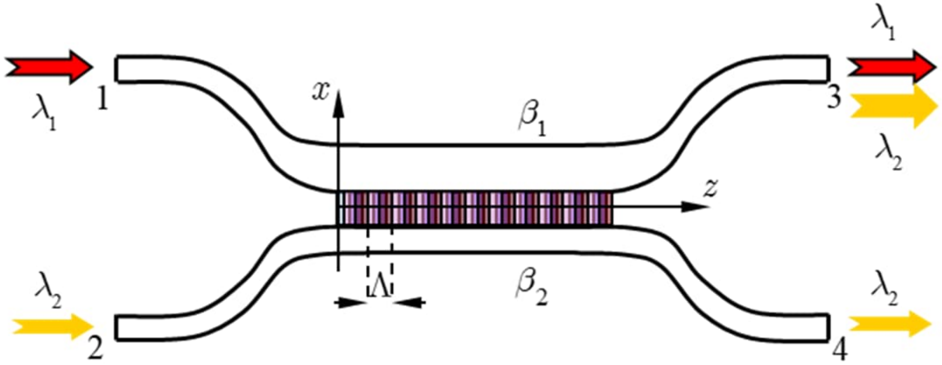




STRAIGHFORWARD ACGC APPLICATIONS

Wideband ADD multiplexor

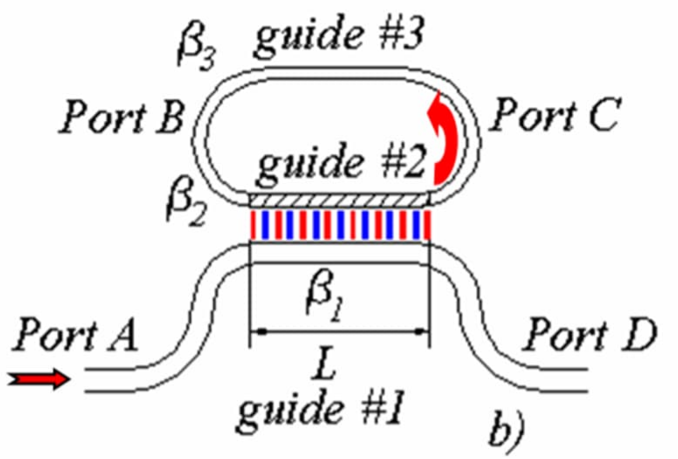
M. Greenberg and M. Orenstein,
PTL **17**, 1450-1452, 2005



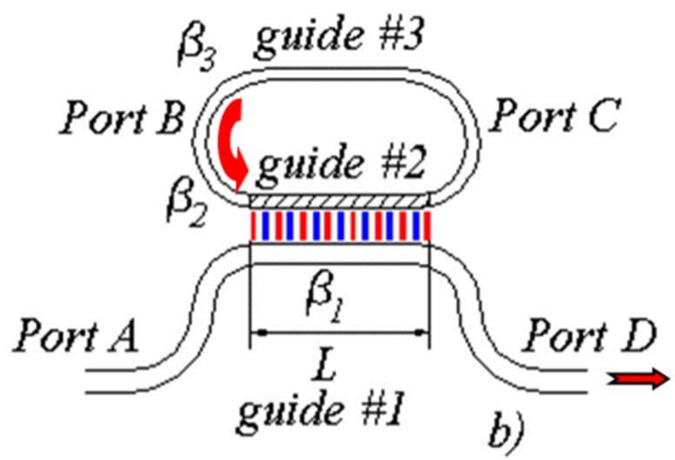
Light trapping in a ring resonator
(a “dynamic memory cell”)

M. Kulishov *et al.*, *OE* **13**, 3567-3578, 2005

grating “switched on”:

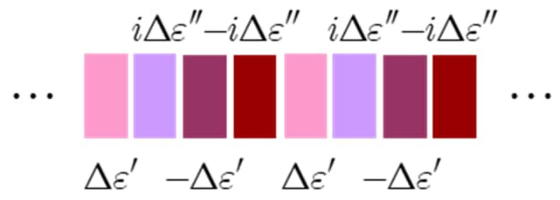


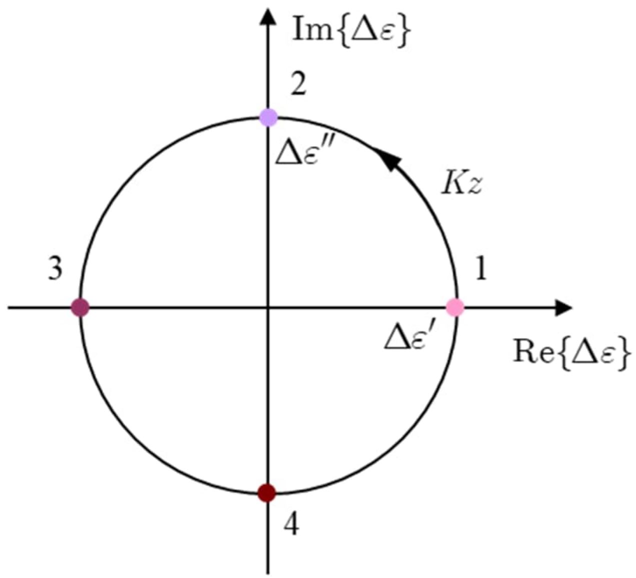
grating “switched off”:



"SSB" MODULATION

For "unidirectional" behaviour, the condition $\Delta\varepsilon' = \Delta\varepsilon''$ is of key importance:

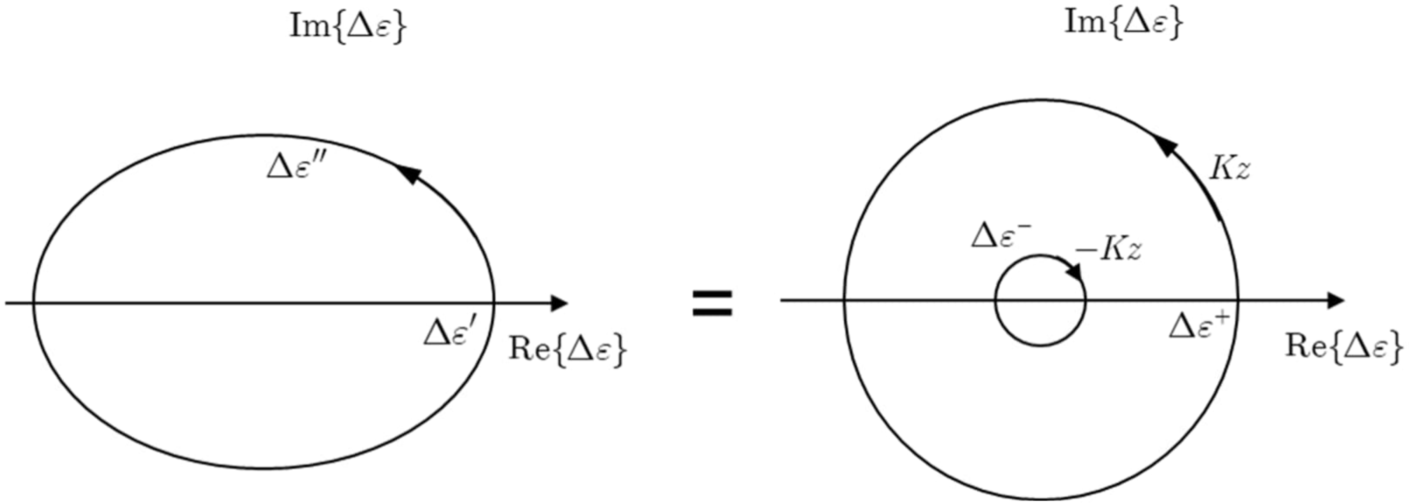




$$\Delta\varepsilon(z) = \Delta\varepsilon' \cos Kz + i\Delta\varepsilon'' \sin Kz$$

$$= \frac{1}{2}\left(\Delta\varepsilon' + \Delta\varepsilon''\right)e^{iKz} + \frac{1}{2}\left(\Delta\varepsilon' - \Delta\varepsilon''\right)e^{-iKz}$$

In the case of $\Delta\varepsilon' \neq \Delta\varepsilon''$ we get



$$\Delta\varepsilon^+ = \left(\Delta\varepsilon' + \Delta\varepsilon''\right) / 2,$$

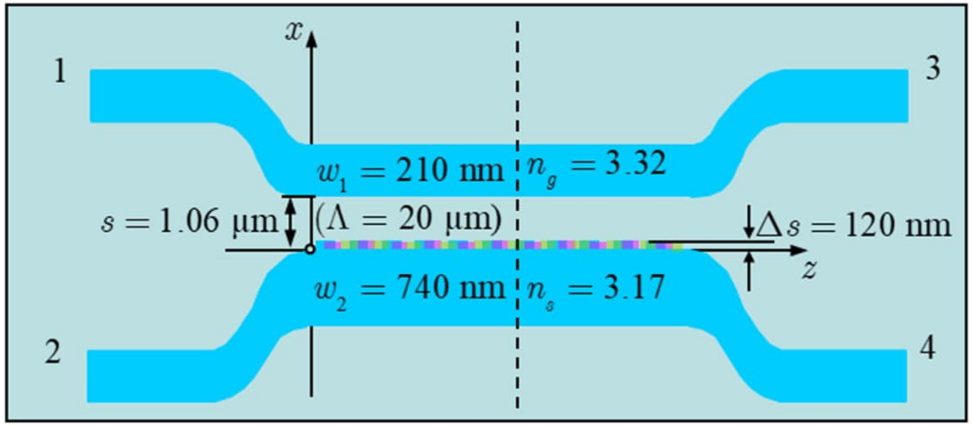
$$\Delta\varepsilon^- = \left(\Delta\varepsilon' - \Delta\varepsilon''\right) / 2.$$

NUMERICAL MODELLING OF ACGC

Method: Bi-directional mode expansion propagation based on harmonic expansion with complex coordinate transformation as a PML (BEXX)

J. Čtyroký, *OQE* **38**, pp. 45-62, 2006; *JLT* **25**, No. 9, pp. 2321-2330, 2007; *JLT* **27**, 2009 (in print)

Basic waveguide structure: asymmetric directional coupler of the InP/GaInAsP type



Asymmetric grating: 24 periods, 4 segments, 5 μm long each

modulation format: alternative,

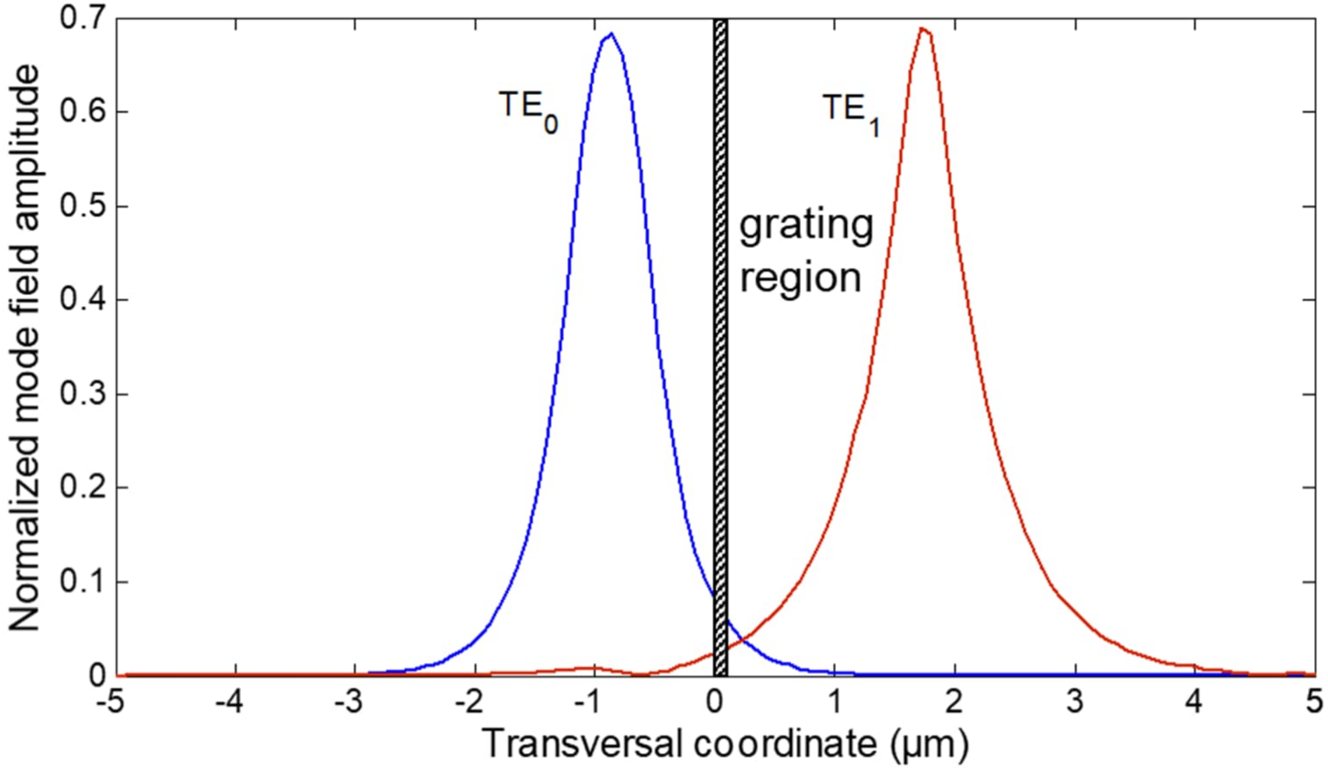
total length of the grating: 24×4×5 μm = 480 μm

central wavelength: λ = 1.532 μm

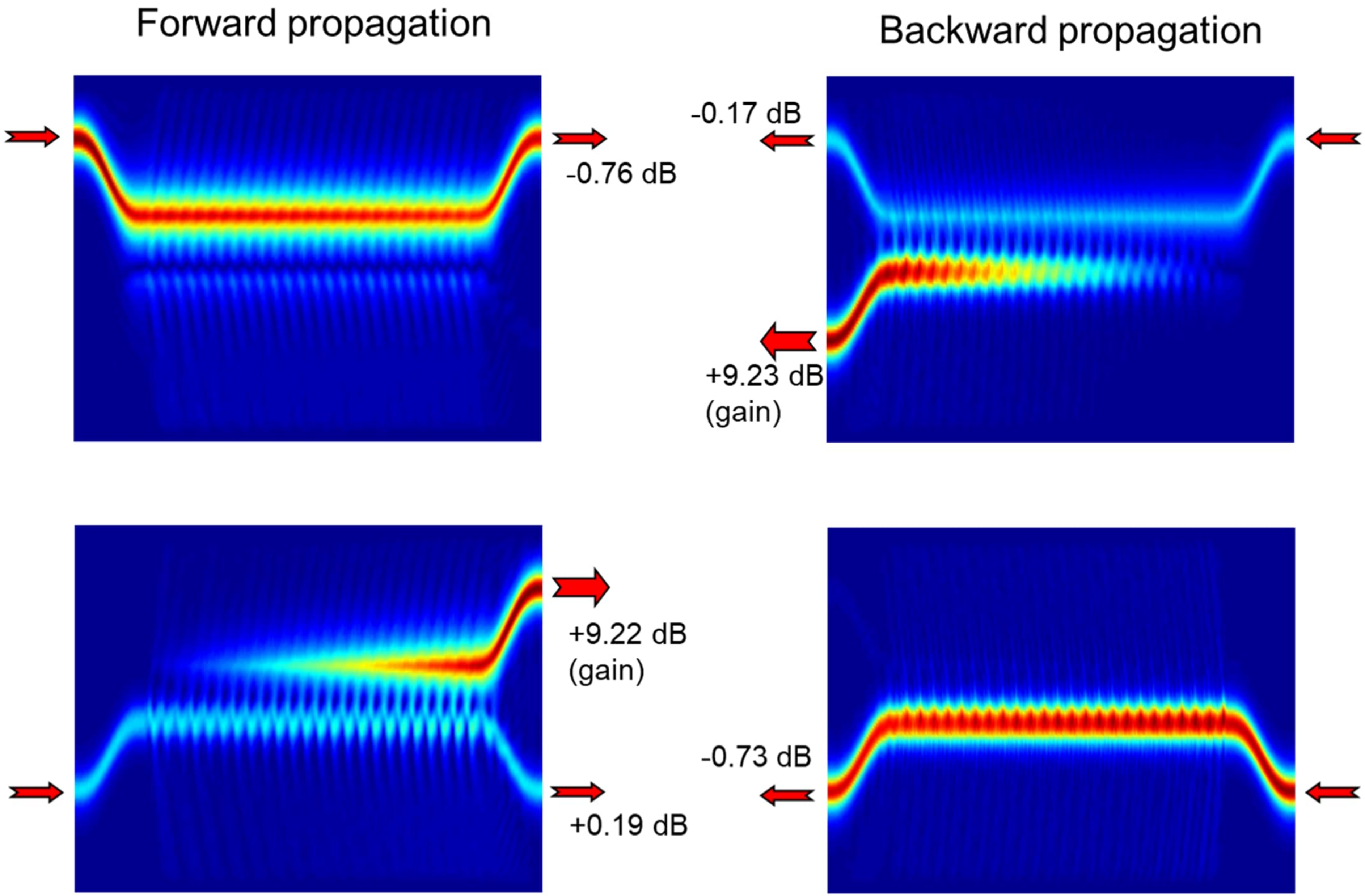
$\Delta\varepsilon' = \Delta\varepsilon'' = 0.48675$ (!!!) (gain/loss ~135 dB/mm!)

“STANDARD” ACGC

Mode field distribution in the central part of the coupler



“STANDARD” ACGC – RESULTS



FORMAL ANALOGY BETWEEN A PHOTONIC WAVEGUIDE AND A POTENTIAL WELL IN QUANTUM MECHANICS

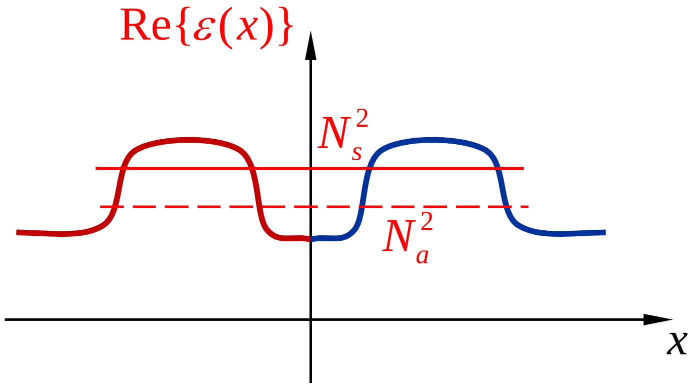
Eigenmode equation for TE modes of a planar waveguide

$$\frac{1}{k_0^2}\frac{d^2E(x)}{dx^2} + \varepsilon(x)E(x) = N^2E(x)$$

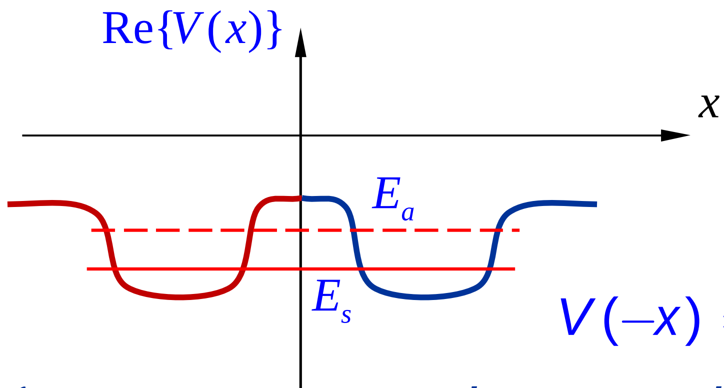
Schrödinger equation for a particle in a 1D potential well

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)$$

mode field distribution	$E(x)$	$\Leftrightarrow$	$\psi(x)$	wave function
wave number	k_0	$\Leftrightarrow$	$\frac{\sqrt{2m}}{\hbar}$	mass; Planck constant
relative permittivity profile	$\varepsilon(x)$	$\Leftrightarrow$	$-V(x)$	potential
effective refractive index	N^2	$\Leftrightarrow$	$-E$	particle energy

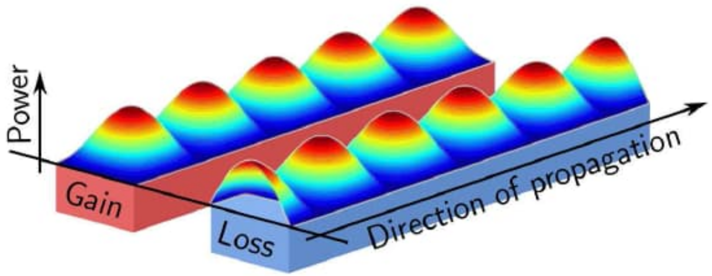


Loss/gain structure: $\varepsilon(-x) = \varepsilon^*(x)$

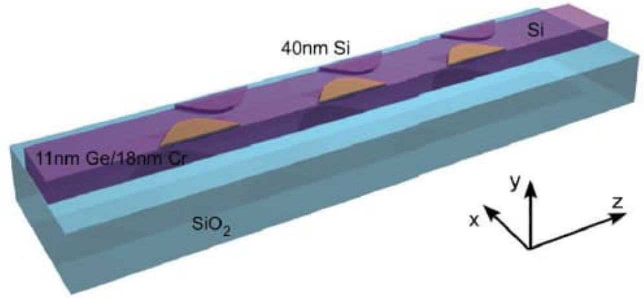


$V(-x) = V^*(x)$

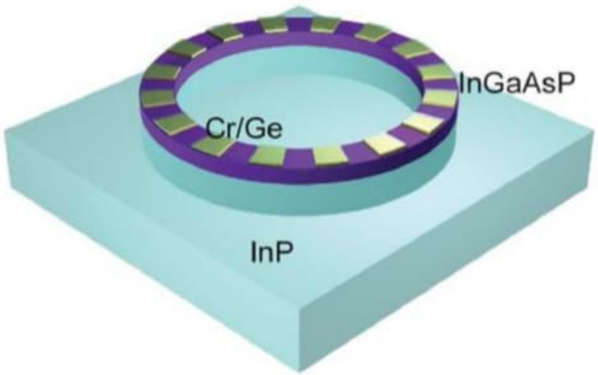
"$\mathcal{PT}$ symmetry": *complex potential(!)*,



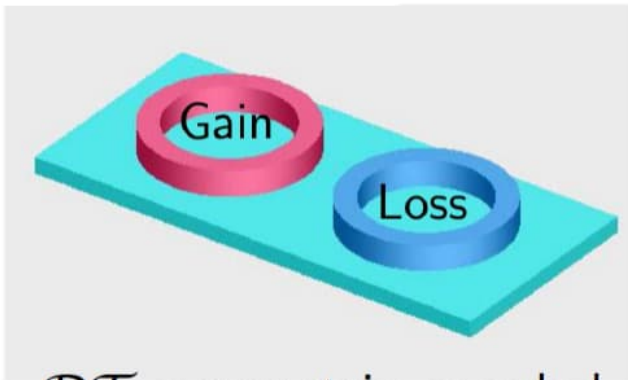
$\mathcal{PT}$-symmetric
coupled waveguide [1.34]



$\mathcal{PT}$-symmetric
waveguide grating [1.74]



$\mathcal{PT}$-symmetric
ring grating [1.61]

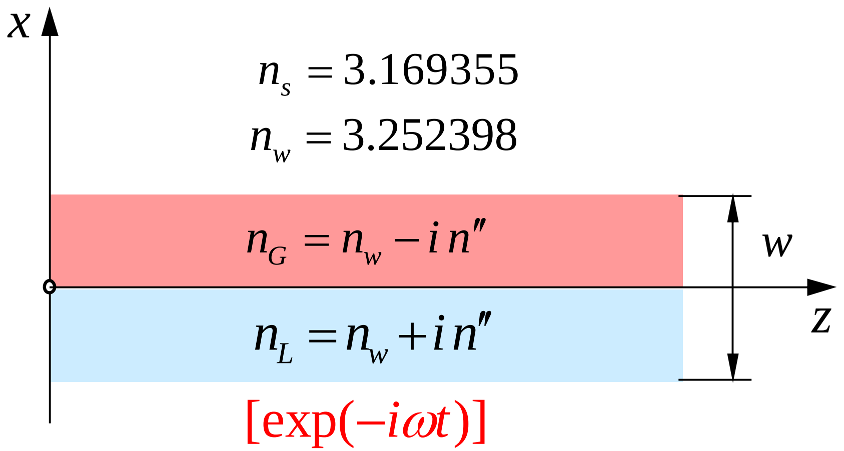


$\mathcal{PT}$-symmetric coupled
microresonators [1.11,1.58]

1.34: C. E. Rüter et al., *Nat. Phys.* 6, 192-195 (2010).
1.61: L. Feng et al., *Science* 346, 972-975 (2014).
1.74: L. Feng et al., *Science* 333, 729-733 (2011).
1.11: L. Chang et al., *Nat. Photonics* 8, 524-529 (2014).
1.58: H. Hodaei et al., *Science* 346, 975-978 (2014).

WAVEGUIDE STRUCTURE WITH LOSS/GAIN: HISTORICAL REMARKS

≈ 1995: COST 240 Action: Loss/gain waveguide modelling task by H. P. Nolting (HHI)
(aimed at benchmarking of BPM methods)

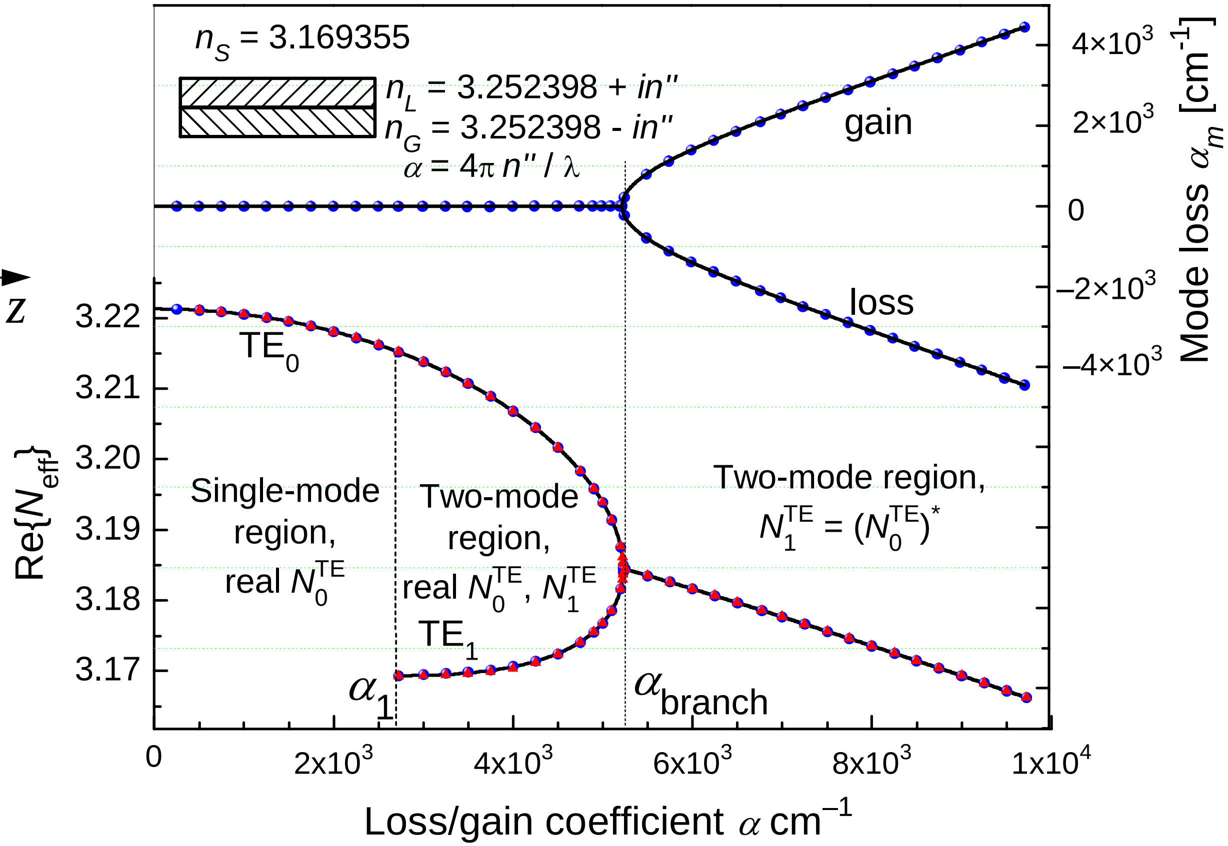


$w = 1$ µm,

$$n'' = \frac{\alpha}{2k_0} = \frac{\lambda}{4\pi}\alpha \times 10^{-4} \text{ [–; µm, cm}^{-1}\text{]},$$

$\lambda = 1.55$ µm,

$\alpha \ldots$ "loss/gain coefficient" [cm^{-1}]



1. H.-P. Nolting, G. Sztefka, J. Čtyroký, "Wave Propagation in a Waveguide with a Balance of Gain and Loss," Integrated Photonics Research '96, Boston, USA, 1996, pp. 76-79.
2. G. Guekos, *Ed.*, Photonic Devices for telecommunications: how to model and measure. Berlin: Springer, 1998, pp. 76-78.

DISPERSION EQUATION (TE modes)

$$\Phi(N,\alpha) = \gamma_G\left[\gamma_S\cos(k_0\gamma_G w) - \gamma_G\sin(k_0\gamma_G w)\right]\left[\gamma_S\sin(k_0\gamma_L w) + \gamma_L\cos(k_0\gamma_L w)\right] + \\ + \gamma_L\left[\gamma_S\cos(k_0\gamma_L w) - \gamma_L\sin(k_0\gamma_L w)\right]\left[\gamma_S\sin(k_0\gamma_G w) + \gamma_G\cos(k_0\gamma_G w)\right] = 0$$

$$\gamma_S = \sqrt{N^2 - n_s^2},\ \gamma_L = \sqrt{n_L^2 - N^2},\ \gamma_G = \sqrt{n_G^2 - N^2},\ k_0 = \frac{2\pi}{\lambda}$$

"Exceptional" point: $\dfrac{dN}{d\alpha} \to \infty.$

Since $\Phi(N,\alpha) \equiv 0$, $\dfrac{\partial\Phi}{\partial N}\dfrac{\partial N}{\partial\alpha} + \dfrac{\partial\Phi}{\partial\alpha} = 0 \quad\Rightarrow\quad \dfrac{\partial N}{\partial\alpha} = -\dfrac{\partial\Phi}{\partial\alpha}\Big/\dfrac{\partial\Phi}{\partial N} \to \infty \quad\Rightarrow\quad \dfrac{\partial\Phi}{\partial N} = 0.$

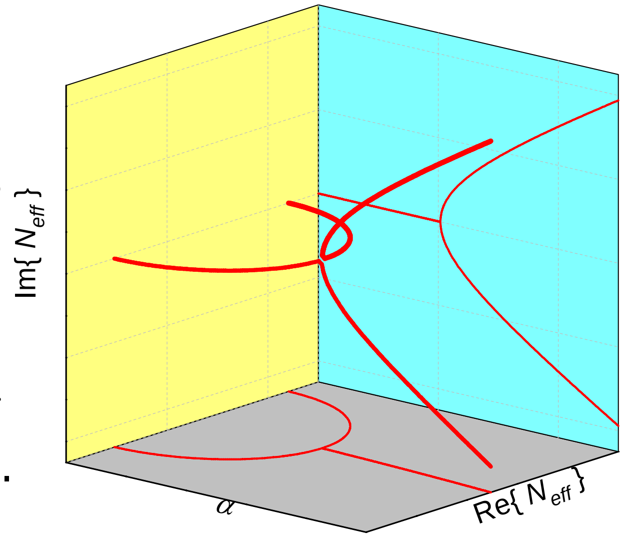
Exceptional point is given by the simultaneous solution of the following two equations:

$$\Phi(N_B,\alpha_B) = 0, \quad \Phi'_N = \frac{d\Phi(N_B,\alpha_B)}{dN} = 0$$

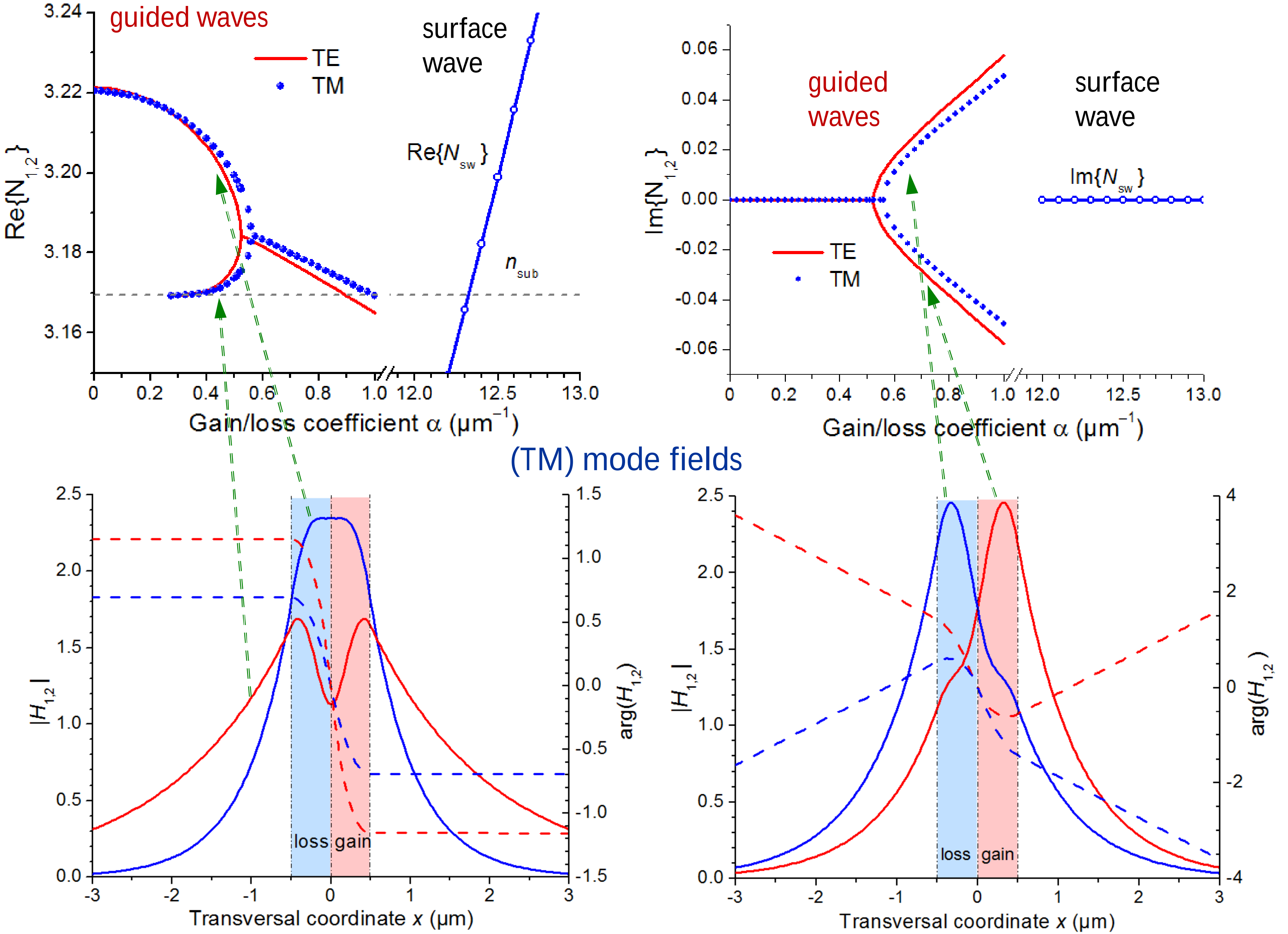
Taylor expansion of $\Phi(N,\alpha)$ in the vicinity of N_B, α_B sounds

$$\Phi(N,\alpha) \approx \Phi'_a(\alpha-\alpha_B) + \frac{1}{2}\Phi''_N\left(N - N_B\right)^2 \overset{!}{=} 0$$

from which it follows $N \approx N_B \pm iC\sqrt{(\alpha-\alpha_B)}$, $C = \sqrt{\dfrac{2\Phi'_\alpha}{\Phi''_N}}.$

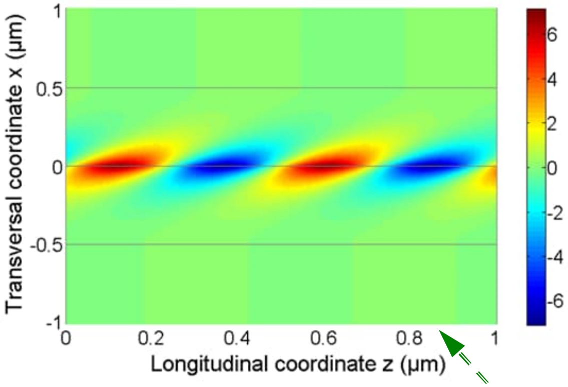


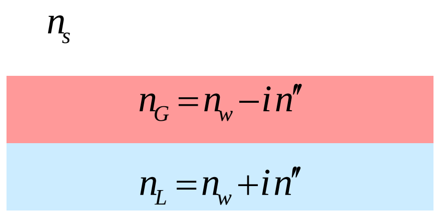
2D ANALYSIS (PLANAR WAVEGUIDES)



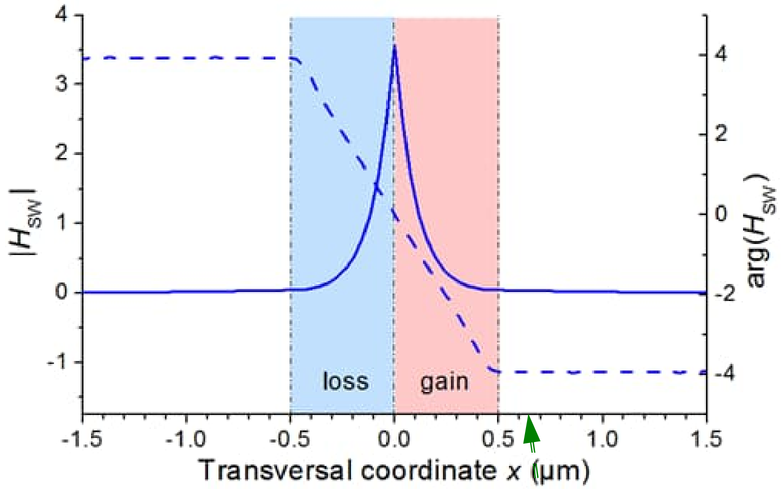
WAVEGUIDE STRUCTURE WITH LOSS/GAIN: SURFACE WAVE

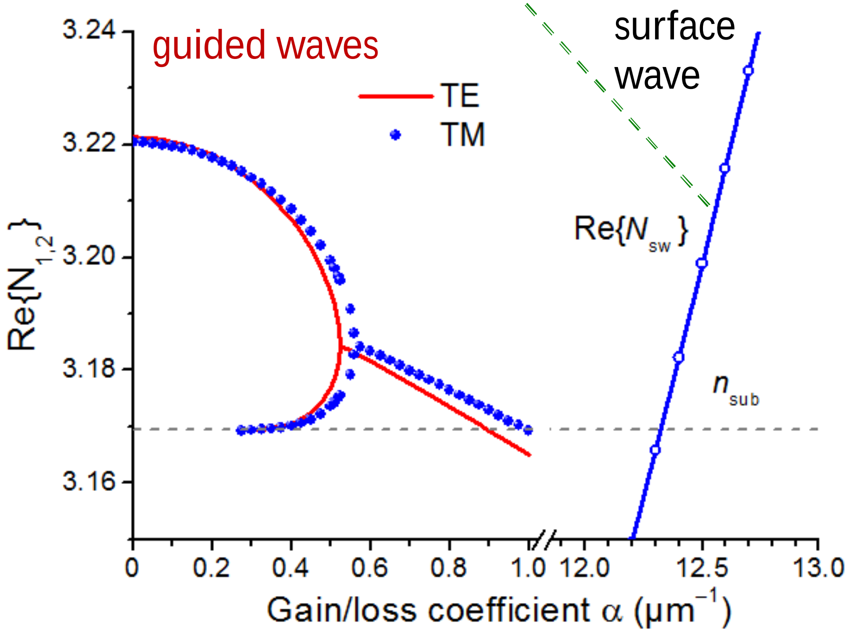
Non-attenuated TM surface wave

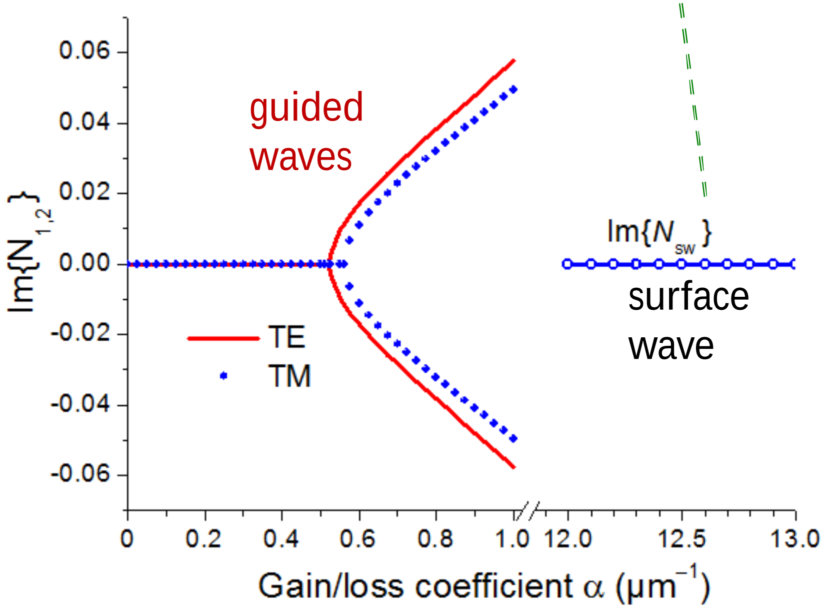




$$N_{SW} = \sqrt{\frac{n_G^2 n_L^2}{n_G^2 + n_L^2}}$$





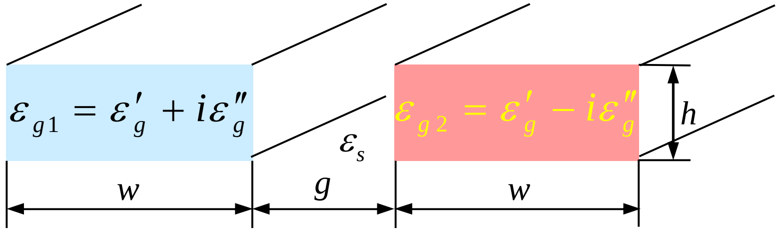


J. Čtyroký *et al.*, "Waveguide structures with antisymmetric gain/loss profile," *Optics Express*, vol. 18, pp. 21585-21593, 2010.

COUPLED WAVEGUIDES WITH LOSS/GAIN

Balanced loss/gain switching:

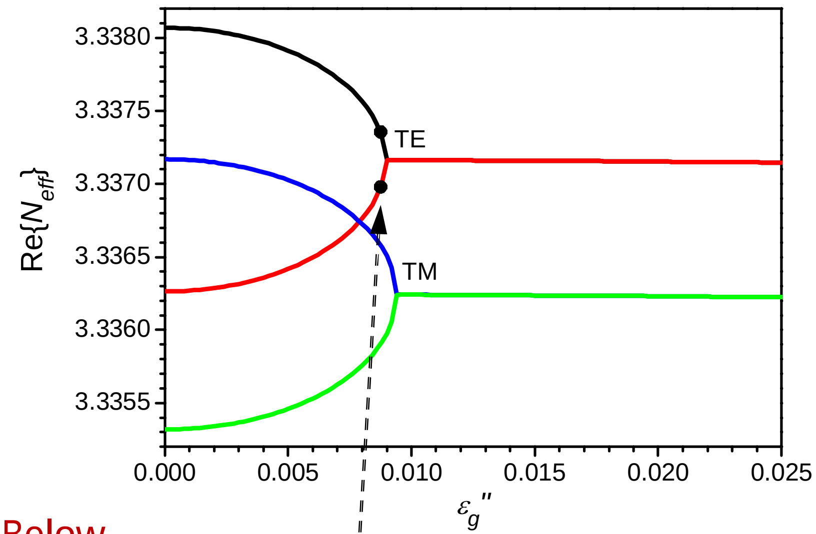
$$\varepsilon(-x,y)=\varepsilon^{*}(x,y)$$

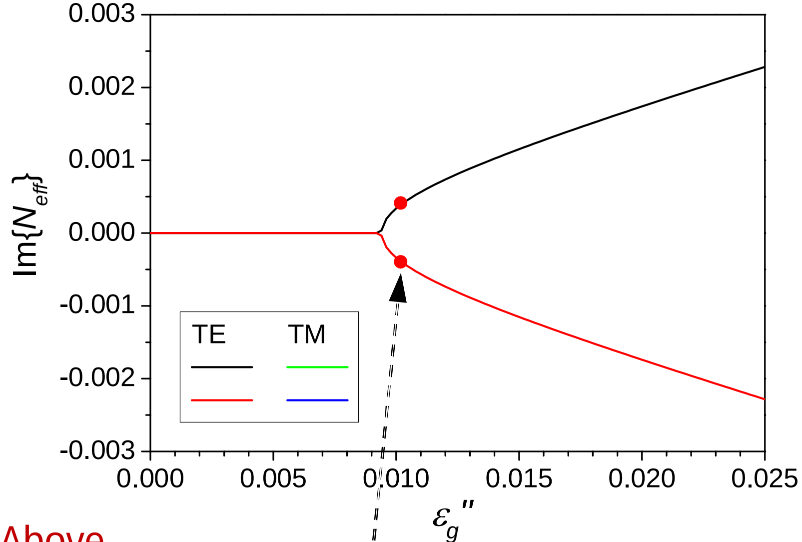


$\varepsilon_s=10.89,\quad \varepsilon_g'=11.56$

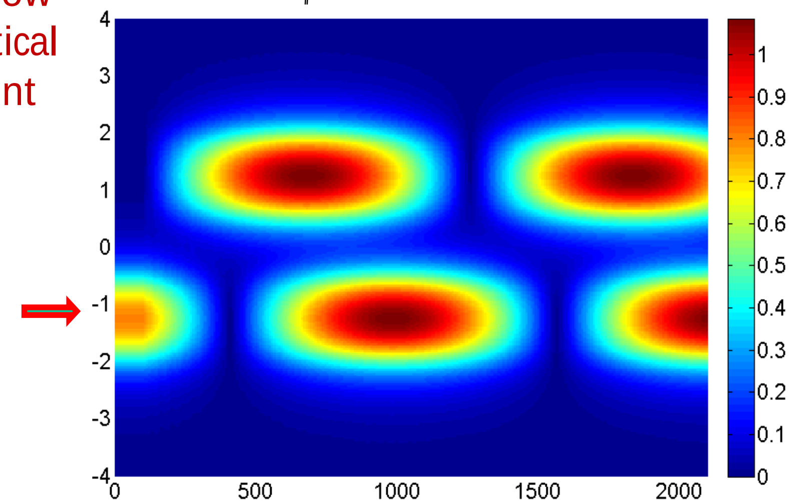
$w=1.5\ \mu m,\ h=0.75\ \mu m,$

$g=1\ \mu m,\quad \lambda=1.55\ \mu m.$





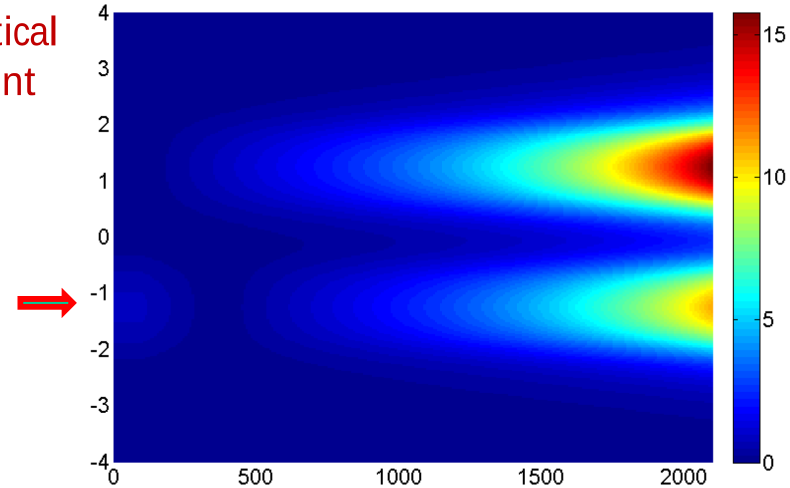
Below critical point



gain channel

loss channel

Above critical point

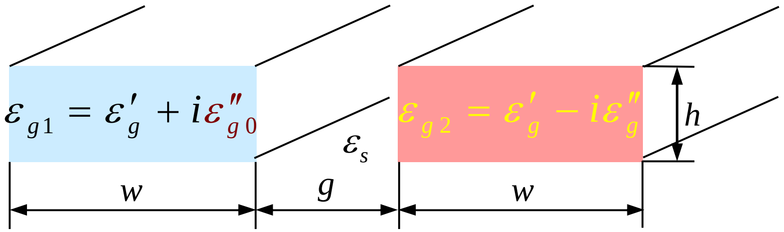


C. E. Rüter *et al.* "Observation of parity–time symmetry in optics," *Nature Physics*, vol. 6, pp. 192-195, 2010.

COUPLED WAVEGUIDES WITH LOSS/GAIN

Fixed loss/variable gain switching:

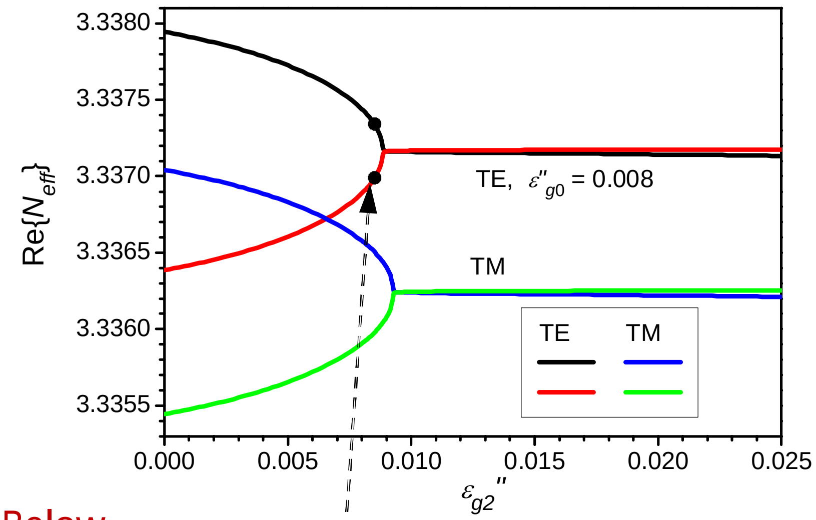
$$\varepsilon(-x,y) \neq \varepsilon^*(x,y)$$

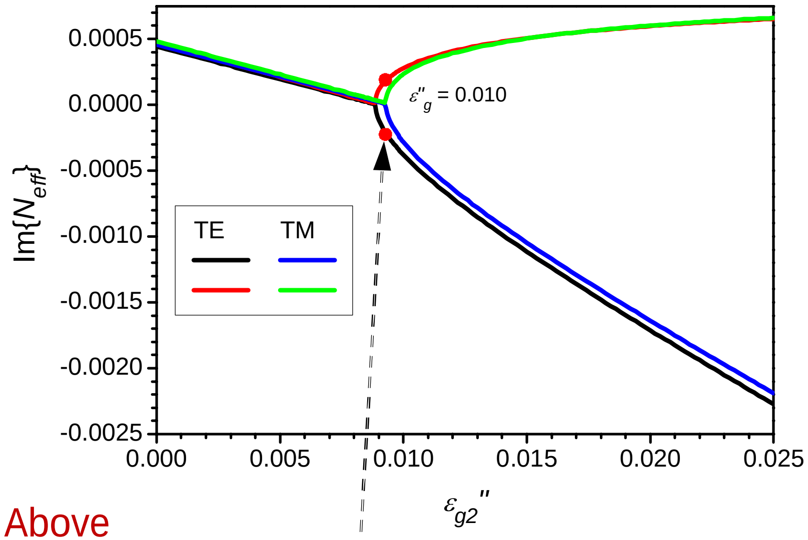


$\varepsilon_s = 10.89, \quad \varepsilon'_g = 11.56$

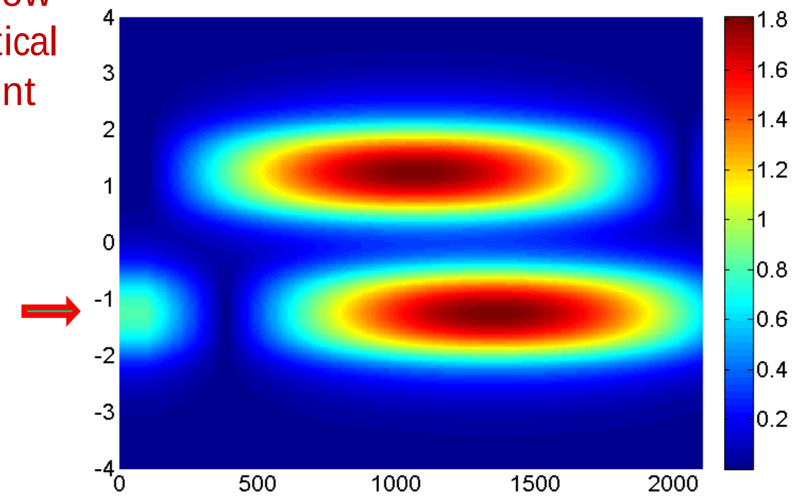
$w = 1.5\ \mu m, \quad h = 0.75\ \mu m,$

$g = 1\ \mu m, \quad \lambda = 1.55\ \mu m.$





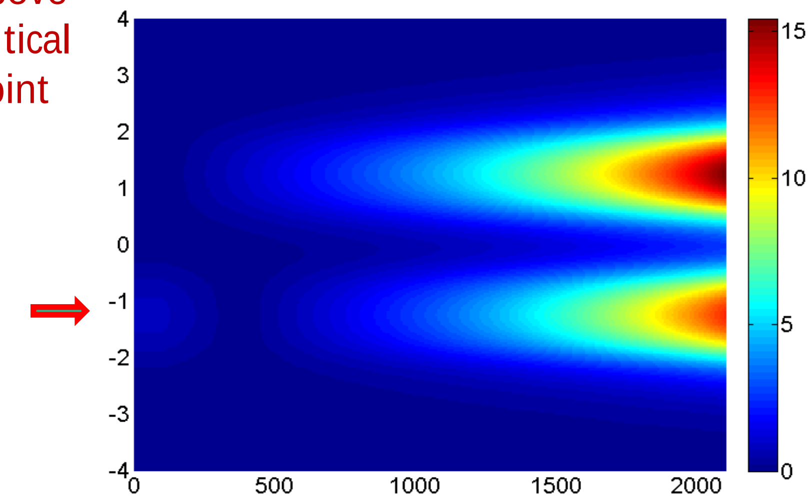
Below critical point



gain channel

loss channel

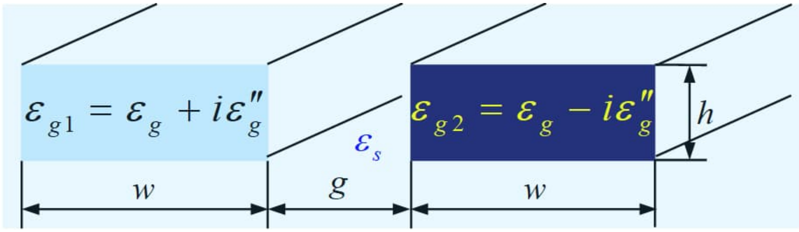
Above critical point



COUPLED WAVEGUIDES WITH LOSS/GAIN

Balanced loss/gain with reduced gain:

$$\mathbf{E}(x,y,z)=\mathbf{e}(x,y)\exp(ik_0 Nz)$$



$\varepsilon_s=10.89+i\varepsilon''_b,\quad \varepsilon_g=11.56+i\varepsilon''_b$

$w=1.5$ μm, $h=0.75$ μm,

$g=1$ μm, $\lambda=1.55$ μm.

Rigorous equation for transversal field components: $\Delta_\perp \mathbf{e}_\perp+\nabla_\perp\left[\nabla_\perp(\ln\varepsilon)\cdot\mathbf{e}_\perp\right]+k_0^2(\varepsilon-N^2)\mathbf{e}_\perp=\mathbf{0},$

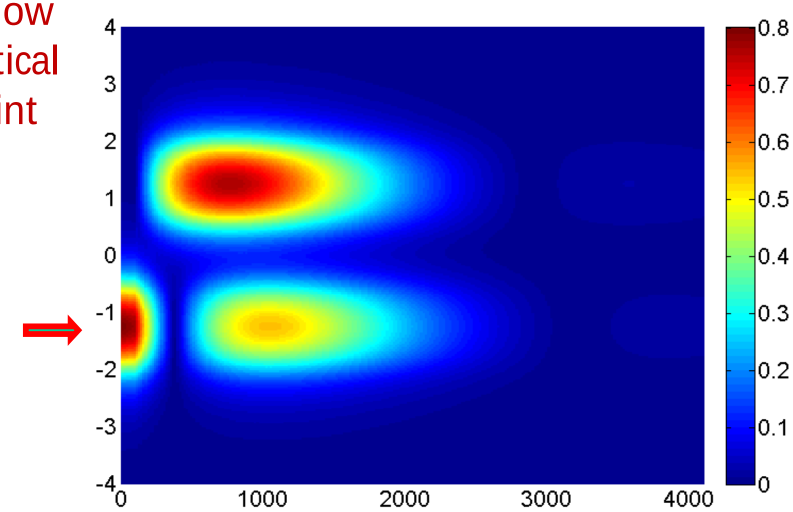
Small uniform permittivity modification: $\varepsilon_1(x,y)=\varepsilon(x,y)+i\varepsilon''_b,\quad |\varepsilon_b|\ll|\varepsilon|$

$$\Delta_\perp \mathbf{e}_\perp+\nabla_\perp\left[\nabla_\perp\left(\ln\varepsilon+\frac{i\varepsilon''_b}{\varepsilon}\right)\cdot\mathbf{e}_\perp\right]+k_0^2\left[\varepsilon+i\varepsilon''_b-(N^2+i\varepsilon''_b)\right]\mathbf{e}_\perp=\mathbf{0};\quad \varepsilon\to\varepsilon+i\varepsilon''_b\ \Rightarrow\ \boxed{N^2\to N^2+i\varepsilon''_b}$$

Uniform background loss can be used to reduce the required gain: $\varepsilon''_{branch}=\pm 0.009$

$\varepsilon''_{g,loss}=0.0105,\quad \varepsilon''_{g,gain}=-0.0065,\qquad \varepsilon''_b=0.002,\qquad \varepsilon''_{g,loss}=0.0115,\quad \varepsilon''_{g,gain}=-0.0075$

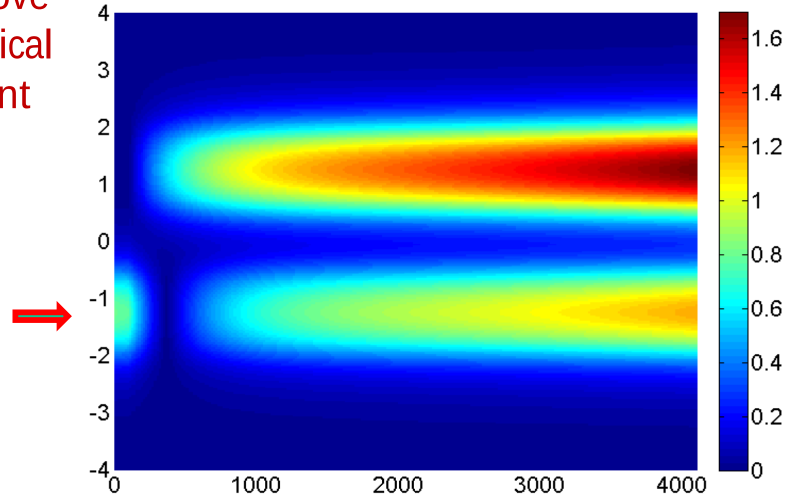
Below critical point



gain channel

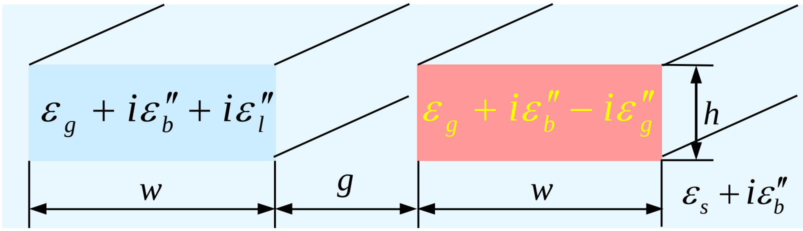
loss channel

Above critical point



COUPLED WAVEGUIDES WITH LOSS/GAIN

Un/balanced loss/gain:



$\varepsilon_s = 10.89, \quad \varepsilon_g = 11.56$

$w = 1.5\ \mu\text{m}, \quad h = 0.75\ \mu\text{m},$

$g = 1\ \mu\text{m}, \quad \lambda = 1.55\ \mu\text{m}.$

Structure with *uniform background loss*, $\varepsilon''_b = 0.002$

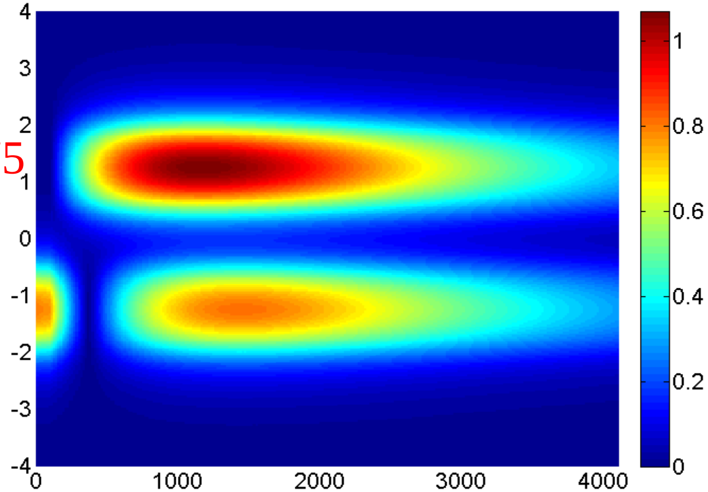
Output *power increase* (from both waveguides!) by *increasing loss* of the lossy channel:

Lower loss, "subcritical" regime

Below critical point

$\varepsilon''_g = -0.0075$

$\varepsilon''_l = 0.0105$



gain channel

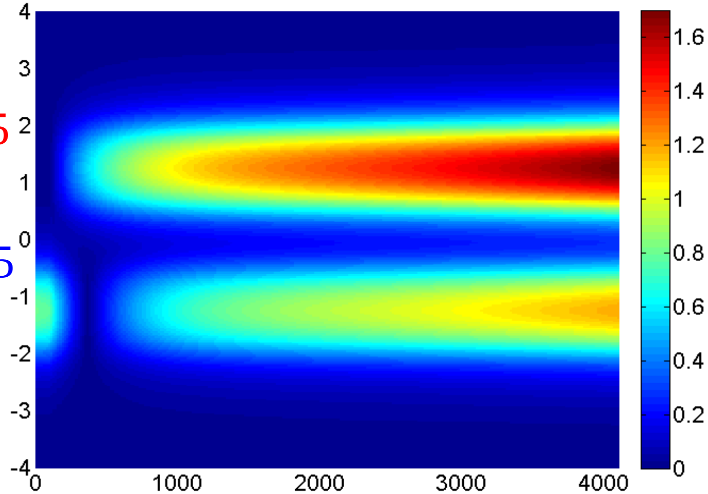
loss channel

Higher loss, "supercritical" regime

Above critical point

$\varepsilon''_g = -0.0075$

$\varepsilon''_l = 0.0115$



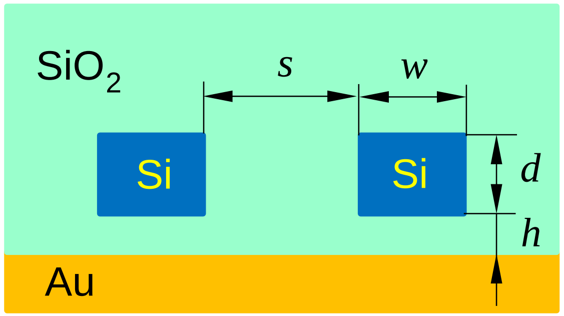
A. Guo *et al.*, "Observation of PT-Symmetry Breaking in Complex Optical Potentials," *Physical Review Letters, vol. 103, no. 9, pp. 093902-1-4, 2009.*

PLASMONIC LOSS/GAIN STRUCTURES

A hypothetic “canonic” (balanced) plasmonic loss/gain structure:

Hybrid dielectric-plasmonic slot waveguide directional coupler *with “tunable metal”*

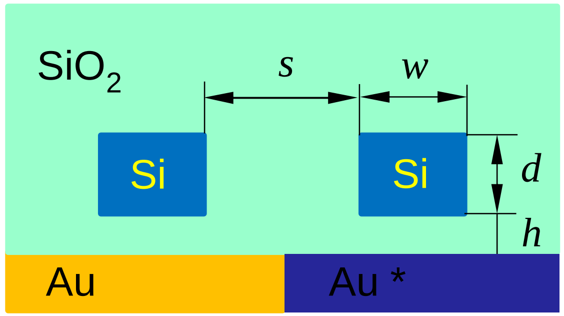
“Passive” structure:



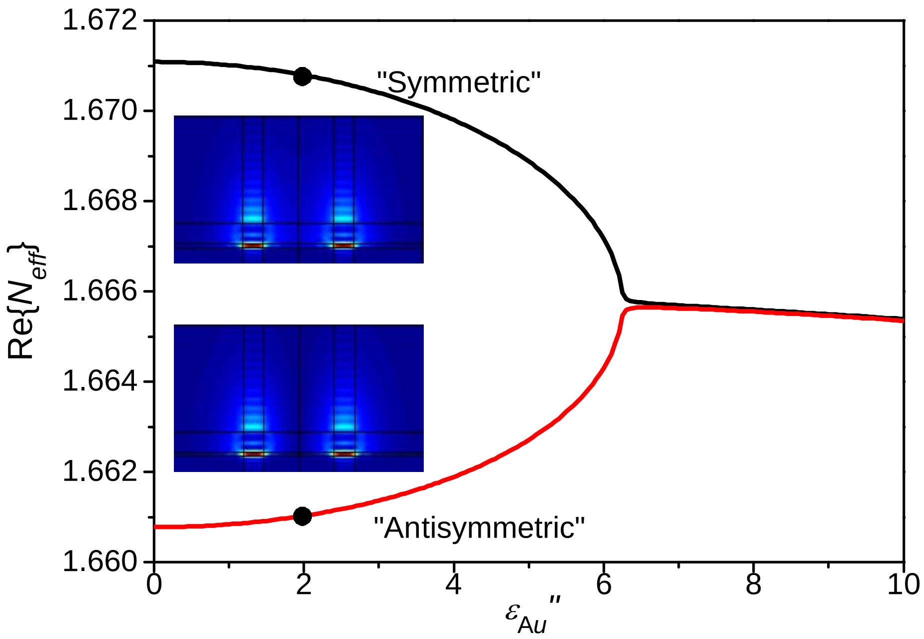
$$\varepsilon(-x, y) = \varepsilon^*(x, y)$$

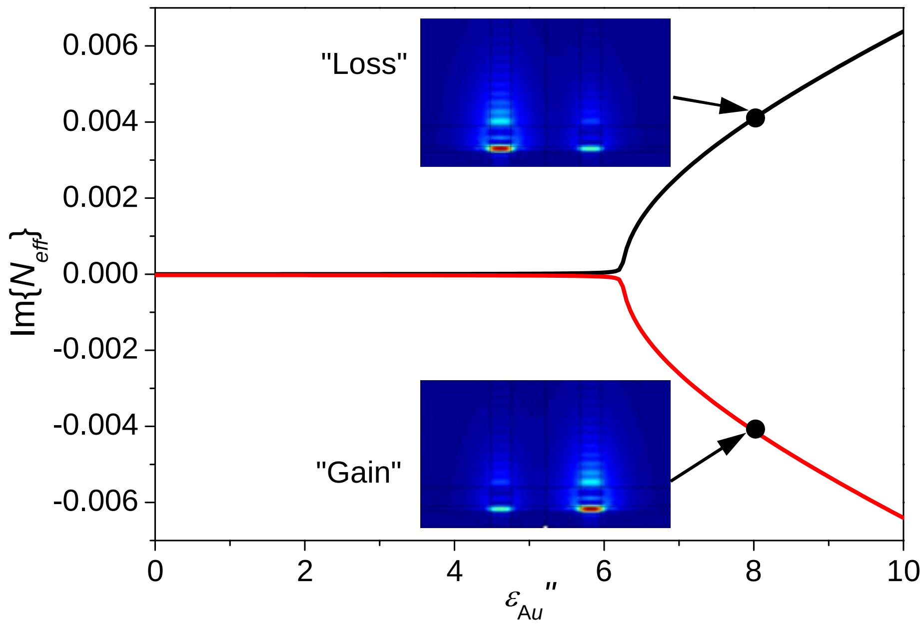
$$w = 300 \text{ nm},$$
$$d = 120 \text{ nm},$$
$$h = 30 \text{ nm},$$
$$s = 1000 \text{ nm},$$
$$\lambda = 1.55\ \mu\text{m}$$

Au with tunable loss / Au* with tunable gain



$$\varepsilon_{Au*} = \varepsilon^*_{Au}$$



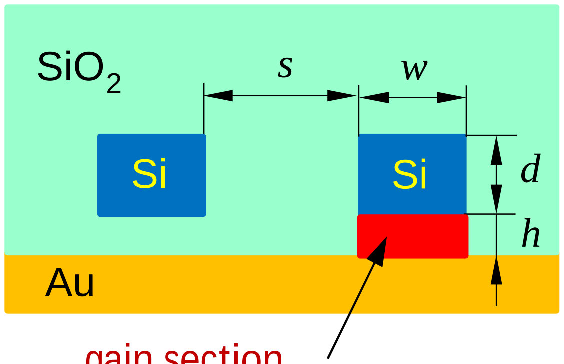


H. Benisty *et al.*, "Implementation of PT symmetric devices using plasmonics: principle and applications," *Optics Express*, vol. 19, pp. 18004-18019, 2011.

PLASMONIC LOSS/GAIN STRUCTURES

A more realistic model of an *unbalanced* plasmonic loss/gain structure:

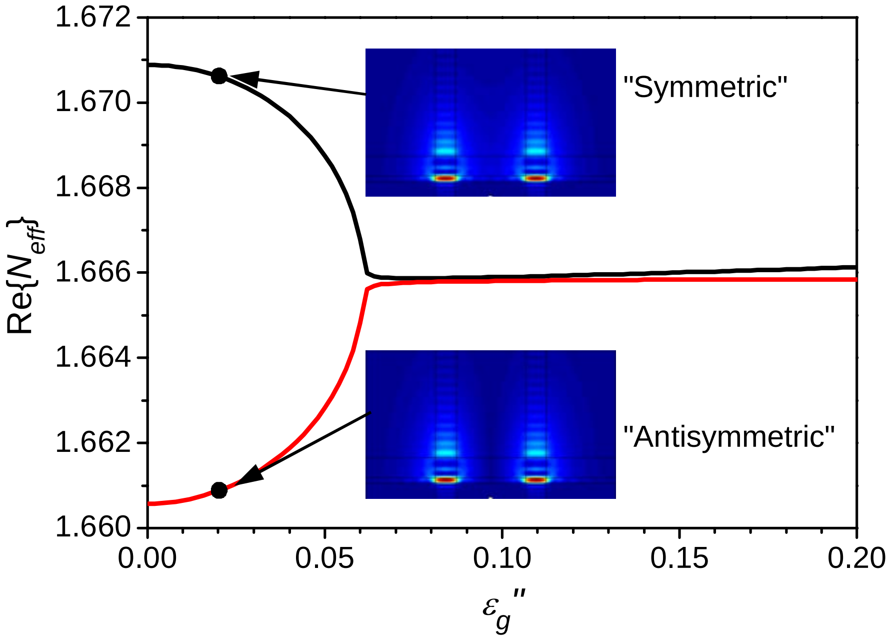
Hybrid dielectric-plasmonic slot waveguide directional coupler with gain section

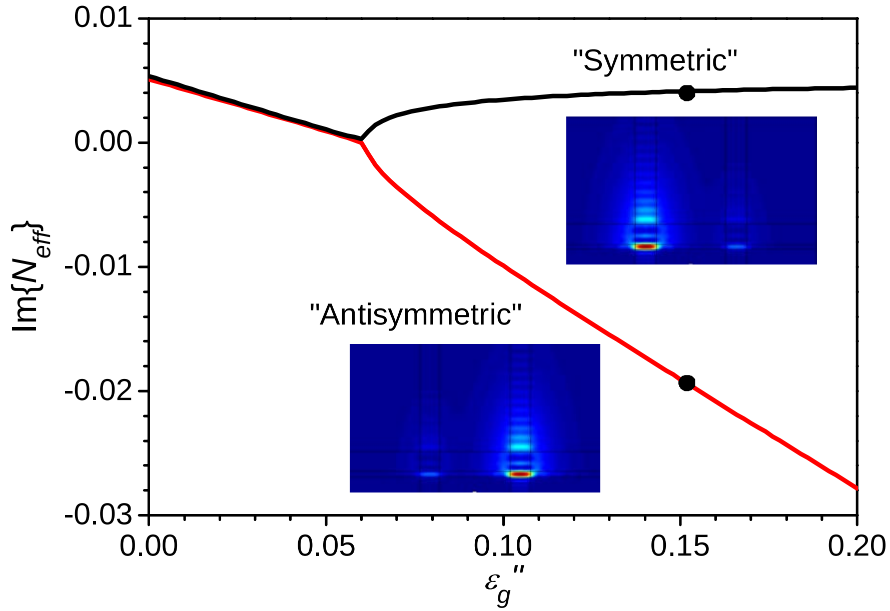


$$w = 300 \text{ nm},$$
$$d = 120 \text{ nm},$$
$$h = 30 \text{ nm},$$
$$s = 1000 \text{ nm}$$

$$\varepsilon(-x, y) \neq \varepsilon^*(x, y)$$

Only gain (ε_g'') in the gain section is now tuned: $\varepsilon_{gain} = \varepsilon_{SiO_2} - i\varepsilon_g''$





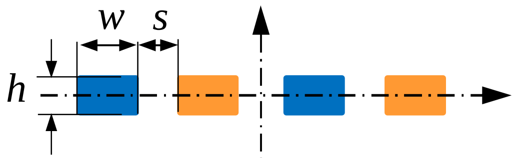
MORE COMPLEX GAIN-LOSS STRUCTURES

Linear arrays of coupled waveguides with loss and gain

(quasi-TE polarization)

$$\varepsilon(-x,y)=\varepsilon^{*}(x,y)$$

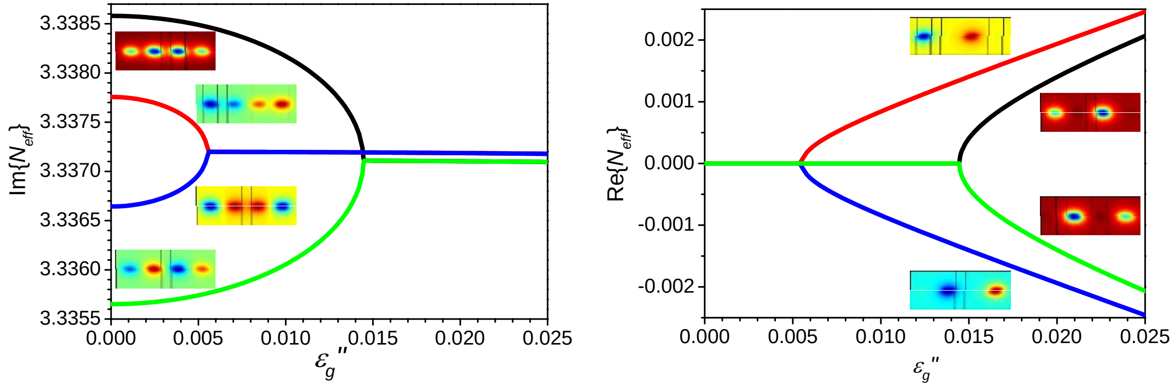
4 coupled channel waveguides



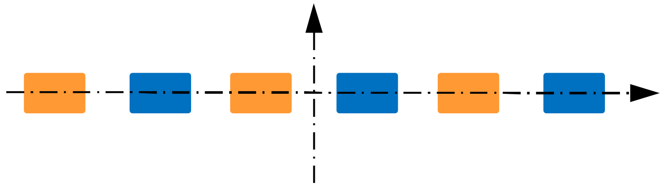
$w = 1.5\ \mu\text{m}$,

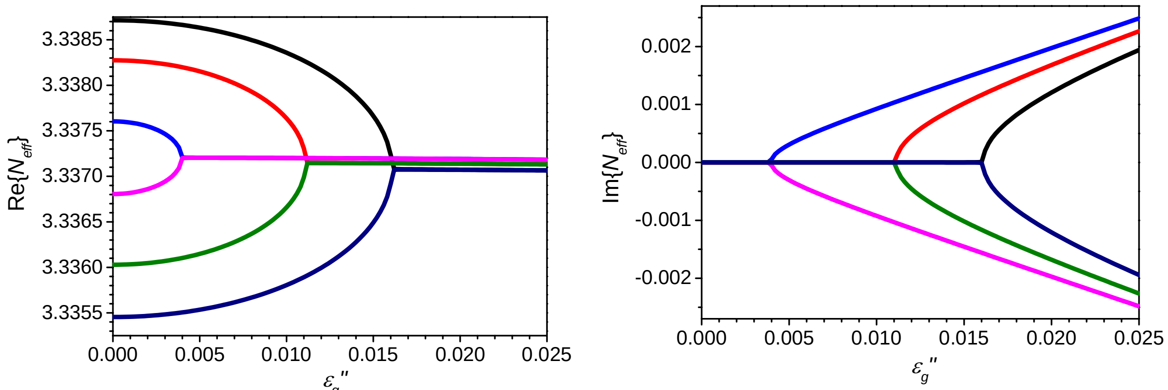
$h = 0.75\ \mu\text{m}$,

$s = 1\ \mu\text{m}$



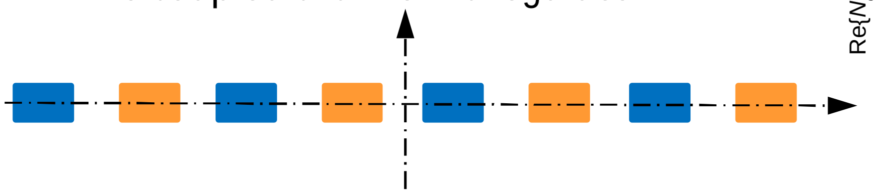
6 coupled channel waveguides

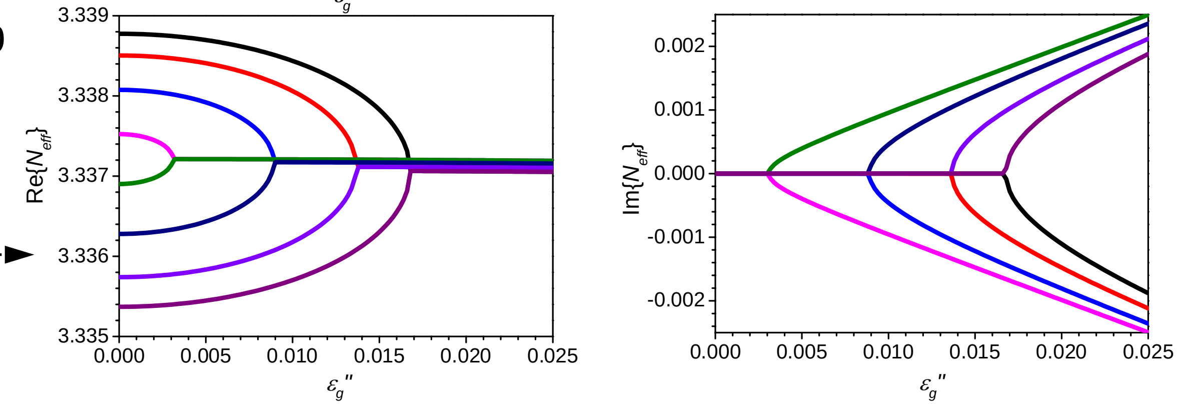




$$\varepsilon_{g1}=11.56+i\varepsilon_g'',\quad \varepsilon_{g2}=11.56-i\varepsilon_g'',\quad \varepsilon_s=10.89$$

8 coupled channel waveguides



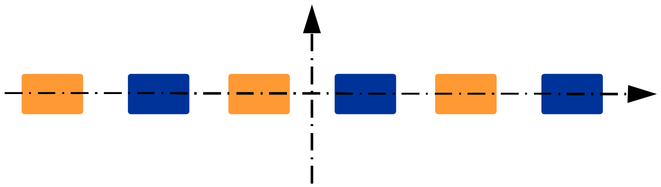


LINEAR ARRAY WITH UNBALANCED LOSS/GAIN

Coupled waveguides with *fixed loss* and *variable gain*

$$\varepsilon(-x,y) \neq \varepsilon^*(x,y)$$

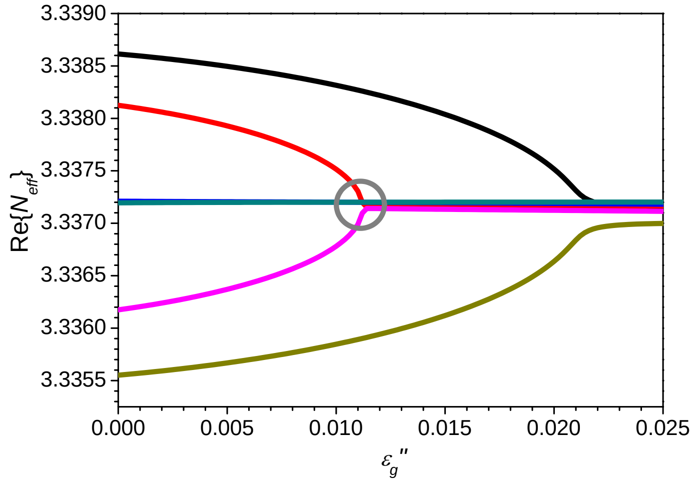
6 coupled channel waveguides

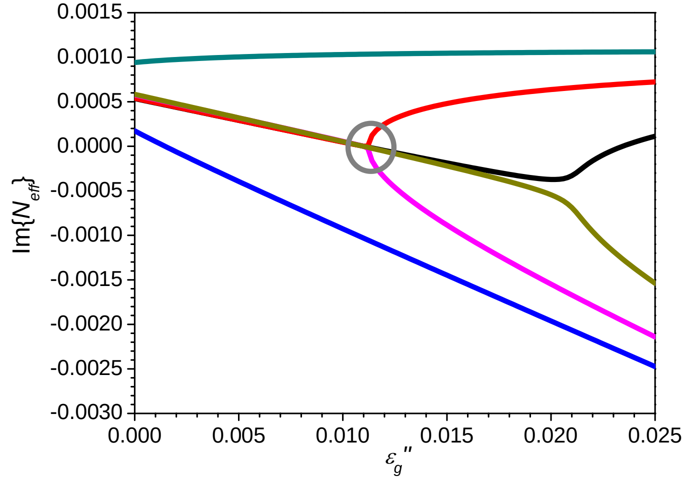


$$\varepsilon_{g1} = 11.56 + 0.011i,$$

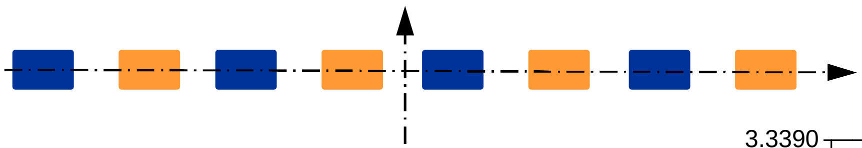
$$\varepsilon_{g2} = 11.56 - i\varepsilon_g'',$$

$$\varepsilon_s = 10.89$$





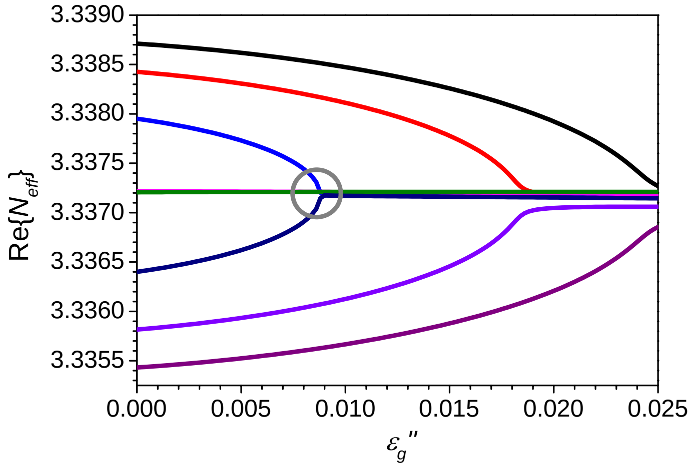
8 coupled channel waveguides

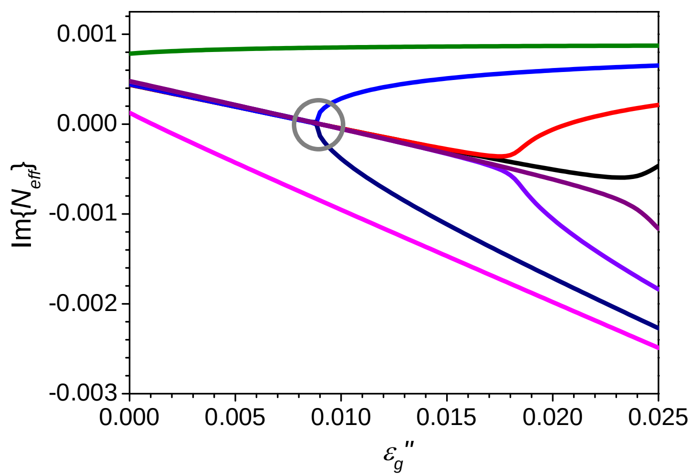


$$\varepsilon_{g1} = 11.56 + 0.009i,$$

$$\varepsilon_{g2} = 11.56 - i\varepsilon_g'',$$

$$\varepsilon_s = 10.89$$





"Switching" by pure gain modulation is feasible also in loss/gain waveguide arrays

MORE COMPLEX GAIN-LOSS STRUCTURES

"Circular" arrays of coupled waveguides with loss and gain

4 waveguides

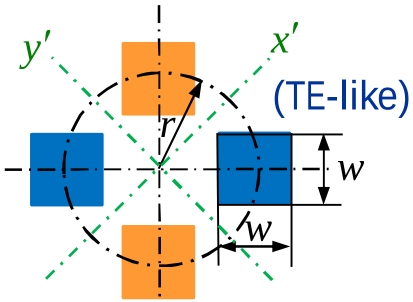
$w = 1\ \mu m$

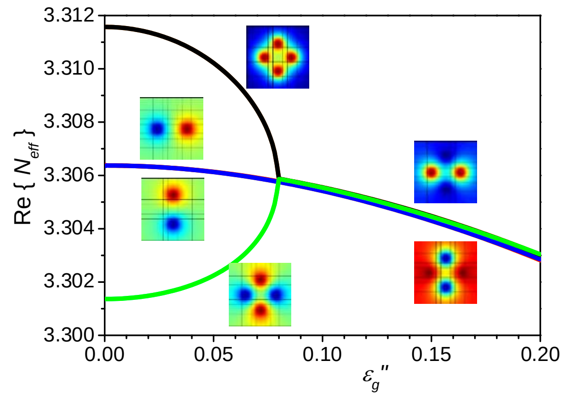
$r = 1.5w$

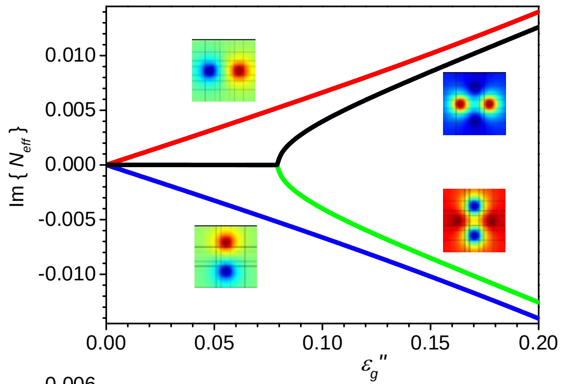
$\varepsilon_{g1} = 11.56 + i\varepsilon_g''$,

$\varepsilon_{g2} = 11.56 - i\varepsilon_g''$,

$\varepsilon_s = 10.89$

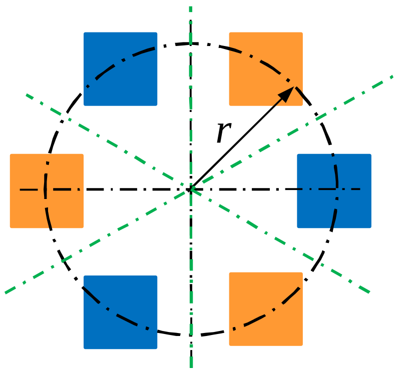


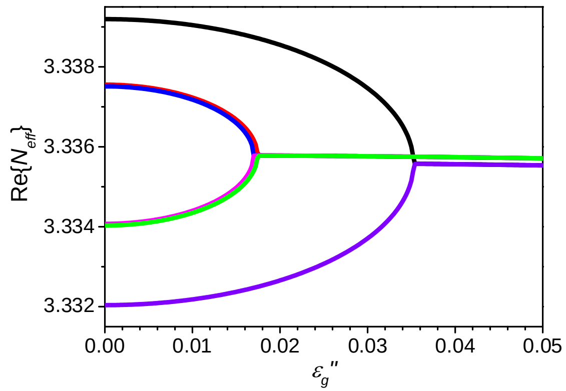


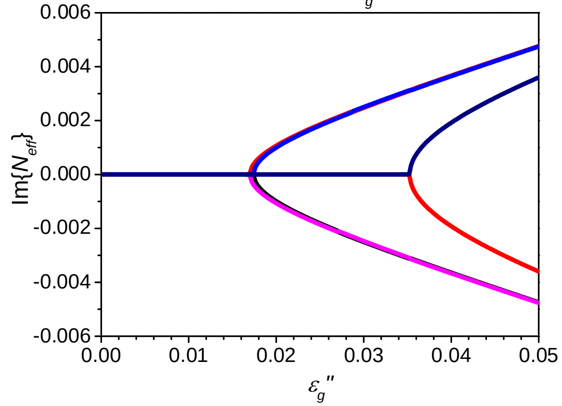


6 waveguides

$r = 2w$

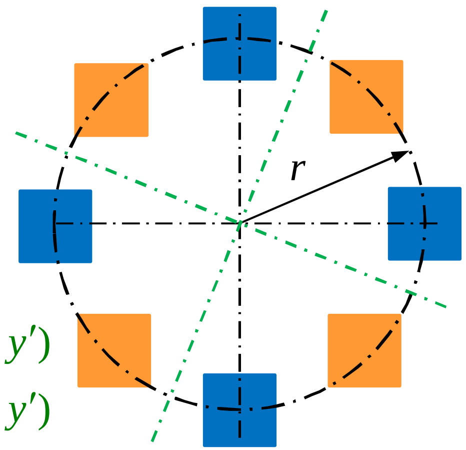






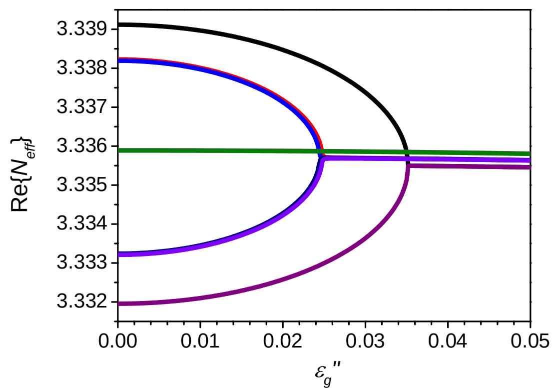
8 waveguides

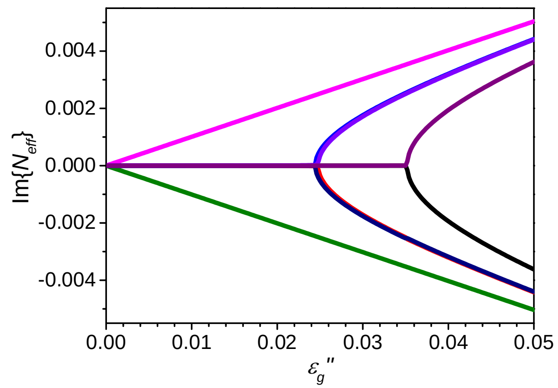
$r = 2.55w$



$\varepsilon(-x', y') = \varepsilon^*(x', y')$

$\varepsilon(x', -y') = \varepsilon^*(x', y')$





CIRCULAR ARRAYS WITH UNBALANCED LOSS/GAIN

Coupled waveguides with *fixed loss* and *variable gain*

6 waveguides

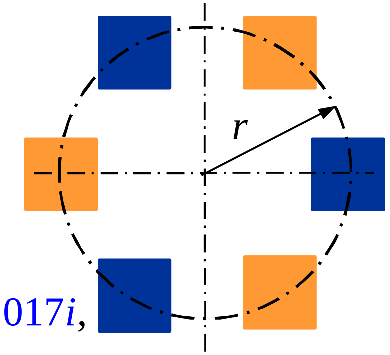
$w = 1\ \mu m$

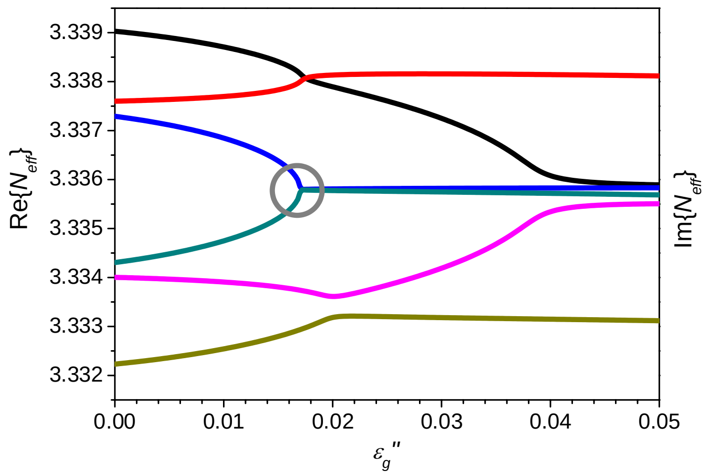
$r = 2w$

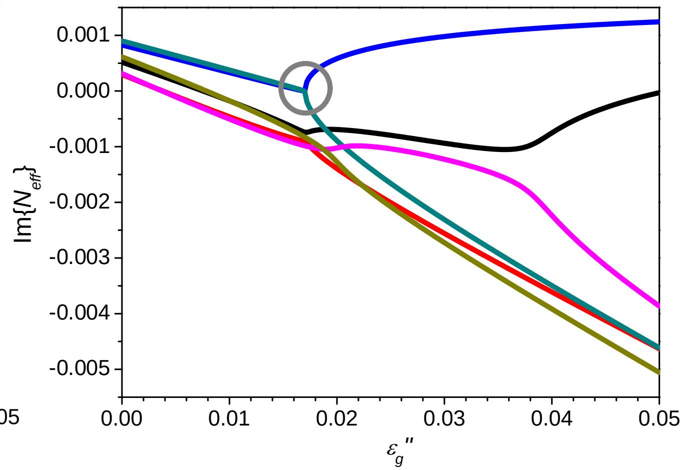
$\varepsilon_S = 10.89$

$\varepsilon_{g1} = 11.56 + 0.017i,$

$\varepsilon_{g2} = 11.56 - i\varepsilon_g''$

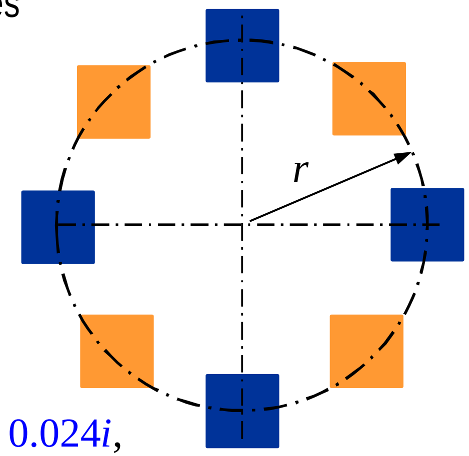






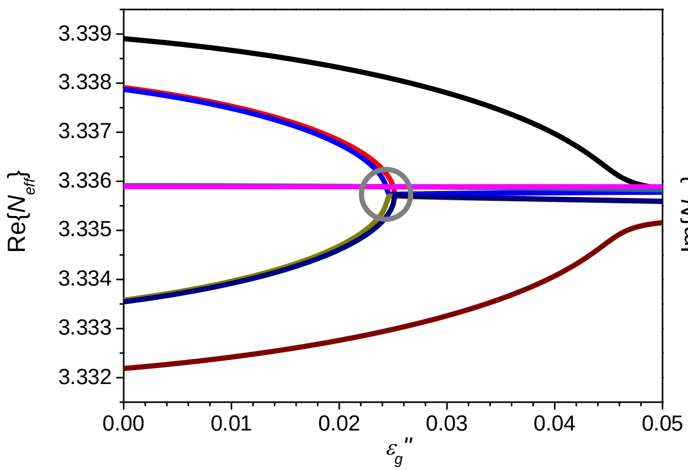
8 waveguides

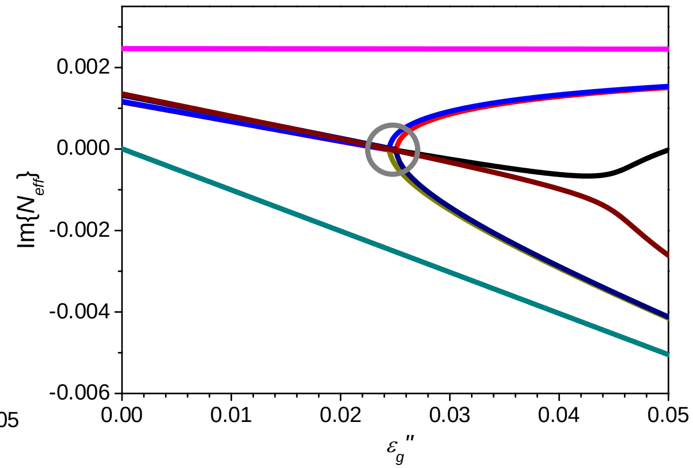
$r = 2.55w$



$\varepsilon_{g1} = 11.56 + 0.024i,$

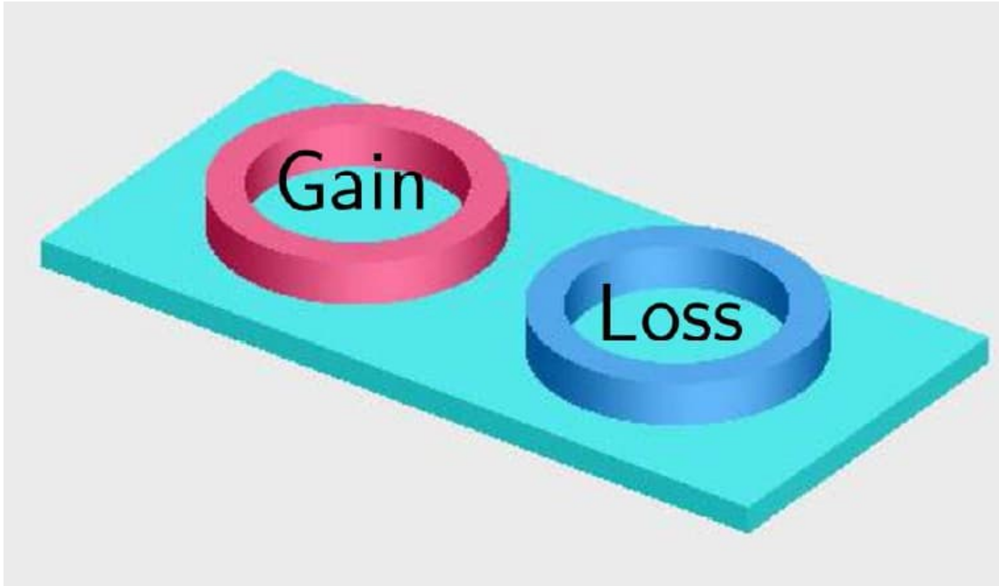
$\varepsilon_{g2} = 11.56 - i\varepsilon_g''$

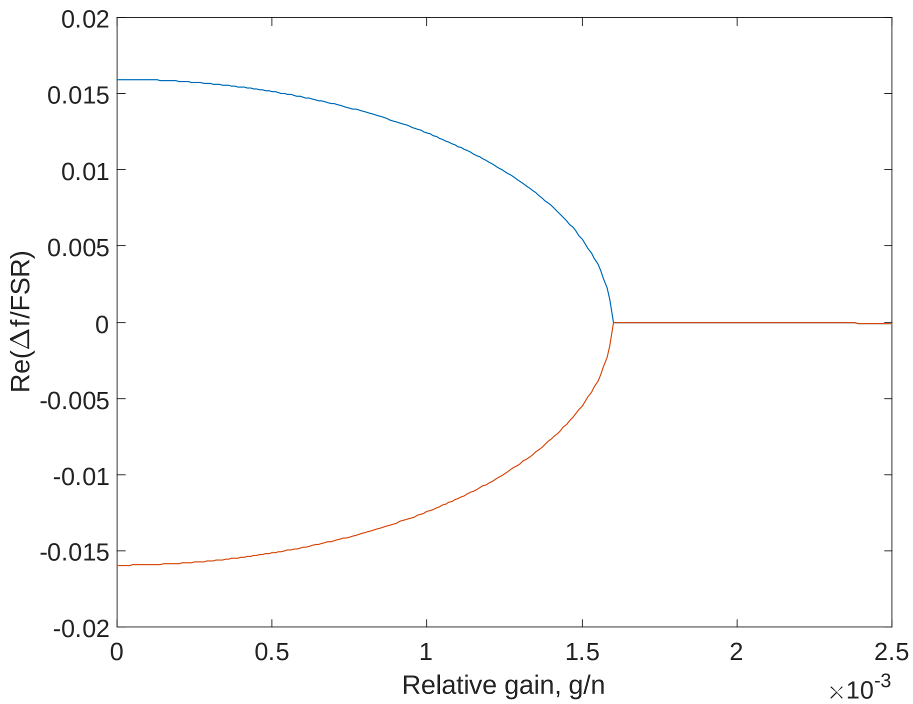


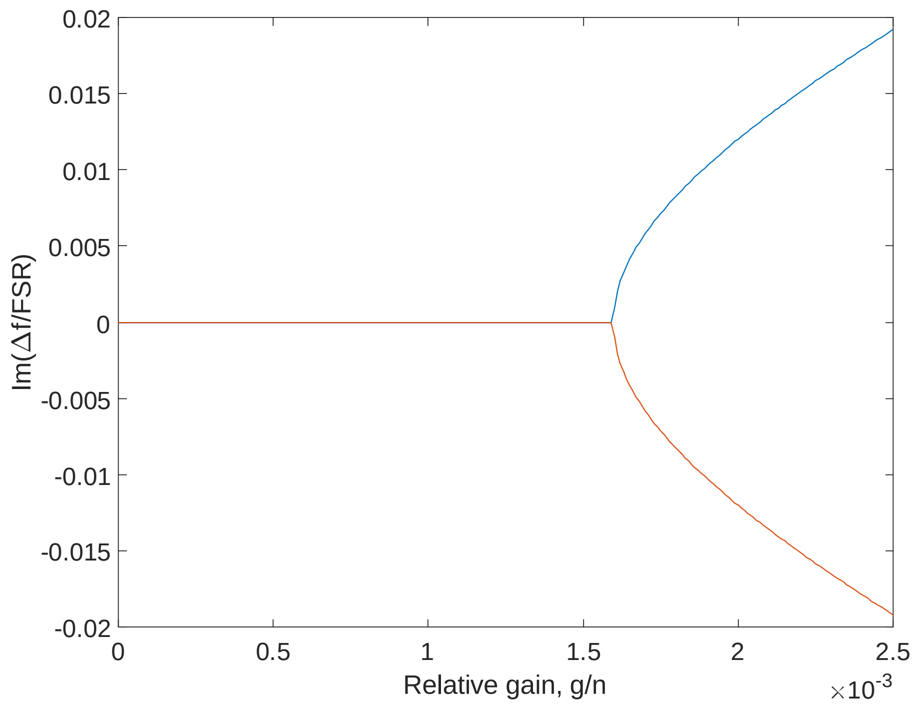


"Switching" by pure gain modulation is feasible also in loss/gain waveguide arrays

RESONANT FREQUENCIES OF A PAIR OF COUPLED *PT* - SYMMETRIC RING RESONATORS







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